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THEORETICAL CONSIDERATIONS RESPECTING THE DEPENDENCE OF WAVE-LENGTH ON PRESSURE WHICH MESSRS. HUMPHREYS AND MOHLER HAVE OBSERVED IN THE ARC-SPECTRA OF CERTAIN ELEMENTS.

By J. WILSING.

THE object of the following article is to show that the interesting results, which have been obtained by Messrs. W. J. Humphreys and J. F. Mohler[1] from their study of the influence of pressure on the wave-lengths of lines in the spectra of the metals, can be explained as an effect of damping of the vibrations to which the emission of light is due, and can be brought into connection with the views which have been developed by Herr v. Lommel[2] in his memoir "Theorie der Absorption und Fluorescenz," and independently by Herr G. Jaumann[3] in a memoir entitled "Zur Kentniss des Ablaufes der Lichtemission."

By raising the pressure in the vessel containing the arc to 15 atmospheres, Messrs. Humphreys and Mohler succeeded in producing displacements of the metallic lines which amounted in exceptional cases to 0.2 tenth-meter, and which were of the same

[1] "Effect of Pressure on the Wave-lengths of Lines in the Arc-Spectra of Certain Elements." This JOURNAL 3, 114, 1896; 4, 175, 249, 1896; 6, 169, 1897.

[2] *Wied. Ann.*, 3, 251, 1878.

[3] *Wied. Ann.*, 53, 832, 1894; this JOURNAL 2, 215, 1895.

order of magnitude as certain systematic differences between the wave-lengths of corresponding solar and metallic lines found by Mr. L. E. Jewell[1] in carrying out his measurements for Rowland's "New Table of Standard Wave-lengths." On increasing the pressure the lines were always displaced toward the red end of the spectrum, that is, their wave-length was increased; while diminution of the pressure below one atmosphere produced a displacement toward the violet end of the spectrum. The amount of the displacement was unequal for lines of different elements. In the case of the different series of lines in the spectrum of any element, it had a definite relation to the number expressing the order of the series. For any line of the same series the change of wave-length was moreover proportional to the change of pressure and to the wave-length itself. Relations were also found between the displacements of lines of elements belonging to the same half of a Mendelejeff's group. Of special significance, however, is the author's remark, that in general displacement was not accompanied by any widening of the lines, such as is usually produced when the density of the vapor is alone increased. On the contrary, the metallic lines have essentially the same width and the same appearance with and without pressure. An explanation of these displacements as the result of an unsymmetrical broadening of the lines is therefore not permissible, although, according to the observations of Ebert,[2] an apparent displacement toward the red end of the spectrum is in general likewise produced, as a consequence of the great broadening which accompanies an increase in the quantity of the radiating vapor. According to Kundt the displacement of the absorption bands of substances dissolved in transparent media is also in the same direction; the greater the dispersion of the non-absorbing medium, the greater is the displacement of the absorption bands. In a later article Mr. Humphreys calls attention to these phenomena, which have a certain analogy to the changes of wave-length caused by pressure, in that the direc-

[1] "The Coincidence of Solar and Metallic Lines." This JOURNAL 3, 89, 1896.

[2] *Wied. Ann.*, 34, 39, 1888.

tion of the displacement is the same in both. He also remarks, in the same place, with reference to v. Lommel's theory of absorption, that the theorem, "An increase of the density or the pressure of a radiating gas causes a broadening of the bright lines in its spectrum and at the same time displaces them toward the red,"[1] necessarily leads to the conclusion that a displacement would be produced by increasing the density of a gas without changing its pressure, which is contrary to experience. This conclusion rests, however, upon a misapprehension which has been corrected by v. Lommel himself in a note on the articles by Jaumann and Prince Galitzin; for the conclusion holds, of course, only when the logarithmic decrement k of the vibration has a value other than zero, or when in general there is damping. In the case of an ideal gas which obeys Mariotte's law, k is, however, according to v. Lommel, always vanishingly small. On the other hand, Humphreys' remark, that displacement *and* broadening must take place simultaneously, according to the theory of v. Lommel, and that this theory is consequently incapable of explaining the phenomena observed in the arc-spectrum when the pressure is increased, since the broadening of the lines must be very considerable relatively to their displacement, is perfectly pertinent in all cases. In fact, the formulæ of v. Lommel require modification or extension, if they are to include the possibility of displacement of lines in consequence of damping, *without* perceptible broadening of the lines.

Von Lommel, like Jaumann, proceeds from the assumption that homogeneous light consists of simple, pendulum-like vibrations. If the vibrations are damped, the corresponding disturbance of the ether is to be regarded as compounded of an infinite number of simple partial vibrations, and therefore capable of being analyzed by Fourier's theorem. But since, in accordance with the fundamental assumption, the resultant obtained by the summation of all these vibrations no longer yields homogeneous light, the broadening of the spectral lines is seen to be a necessary consequence of damping; without, however, implying

[1] *Wied. Ann.*, 3, 267, 1878.

the necessary truth of the inverse theorem, that damping is always present when the spectral lines are broadened.

Von Lommel takes, for the equation of a damped simple harmonic vibration which is also subject to the periodic impulse $-F \sin qt$,

$$m\frac{d^2x}{dt^2} = -K\frac{dx}{dt} - Ax - Bx^2 - Cx^3 - \ldots - F\sin qt,$$

where x denotes the distance of the atom m at the time t from its position of equilibrium. Division by m reduces the equation to the form

$$(1)\quad \frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + p^2x + b\epsilon x^2 + c\epsilon^2 x^3 + \ldots + f\sin qt = 0$$

where

$$\frac{K}{m} = 2k,\ \frac{F}{m} = f,\ \frac{A}{m} = p^2,\ \frac{B}{m} = b\epsilon, \text{ and } \frac{C}{m} = c\epsilon^2.$$

If we neglect the terms containing the small quantity ϵ, which is only of significance in the theory of fluorescence, the integral of the equation is

$$x = M\sin(qt - a) + Ne^{-kt}\sin(rt + \psi),$$
$$r = \sqrt{p^2 - k^2},$$

where k is the coefficient of damping, and $\frac{\pi}{r}$ is the period of the damped main vibration of the atom. The amount of the molecular absorption Δ of the vibration $f \sin qt$ is determined by the expression

$$\frac{m}{2}\left(\int_0^t \left(\frac{dx}{dt}\right)^2 dt - \left[\int_0^t \left(\frac{dx}{dt}\right)^2 dt\right]_{f=0}\right),$$

which gives on integration

$$\Delta = \frac{mf^2\sin^2 a}{16k^2};\quad \tan a = \frac{2kq}{p^2 - q^2}.$$

The absorption of the vibration is therefore greatest when $q = p$, that is, when the period of the excitant radiation coincides with that of the atom vibrating *without* damping. According to the theory of v. Lommel and Jaumann, therefore, damping of

the vibrations can produce only a widening of the dark absorption lines and not a displacement, while according to the observations of Humphreys and Mohler bright and dark lines are equally displaced.

The position in the emission-spectrum of the damped fundamental vibration is determined by the equation $r=\sqrt{p^2-k^2}$, where $\frac{\pi}{r}$ and $\frac{\pi}{p}$ denote the periods of the damped and undamped vibrations respectively. But the maximum of intensity in the widened bright line does not exactly coincide with the place of the main vibration. In order to find the distribution of intensity in the bright line widened by damping, we have to seek the amplitude of the partial vibrations of which the damped vibration is compounded. From the equation

$$e^{-kt}\sin rt=\frac{4kr}{\pi}\int_0^\infty \frac{\mu}{(k^2+r^2+\mu^2)^2-4r^2\mu^2}\sin \mu t\, d\mu$$

we obtain for the amplitude f of the vibration whose period is $\frac{\pi}{\mu}$ the value

$$f=\frac{4kr}{\pi}\frac{\mu}{(k^2+r^2+\mu^2)^2-4r^2\mu^2}.$$

The period $\frac{\pi}{\mu_0}$ which corresponds to the maximum of intensity in the widened bright line is determined by the condition $\left(\frac{df}{d\mu}\right)_{\mu=\mu_0}=0$, therefore by

$$\mu_0^2=\tfrac{1}{3}(p^2-2k^2)+\tfrac{2}{3}\sqrt{p^4-p^2k^2+k^4},$$

or,

$$\mu_0^2=r^2+\tfrac{1}{4}\frac{k^4}{p^2}+\ldots$$

When the damping of the vibration is slight, that is, when $\frac{k^4}{p^2r^2}$ is very small, the maximum of intensity practically coincides with the place of the main vibration r.

Since the period $\frac{\pi}{r}$ of the damped vibration is connected

with the period $\frac{\pi}{p}$ of the undamped vibration by the equation $r = \sqrt{p^2 - k^2}$, it follows, according to v. Lommel's theory, and in agreement with the observations of Humphreys and Mohler, that damping must always produce a displacement of the bright line toward the red end of the spectrum. If, however, we regard $\frac{k}{p}$ as a small quantity of the first order, the amount of the displacement is of only the second order, while the breadth of the line increases very rapidly as k increases. Therefore a perceptible displacement of the bright line cannot take place unless it is accompanied by a very great increase in the width of the line.

The result of the foregoing discussion of v. Lommel's theory of absorption is as follows: the theory is so far in agreement with observation, that it requires a displacement toward the less refrangible end of the spectrum to follow an increased damping of the vibrations. On the other hand it is contradictory to observation in that the dark absorption lines actually suffer the same displacement as the bright lines when the pressure is changed, and in that observation shows no perceptible accompanying increase in the breadth of the lines. These contradictions may, however, be easily removed by an extension of the theory, without impairing the validity of its conclusions when applied to the explanation of the phenomena of fluorescence.

The theory of v. Lommel starts from the assumption of a simple sine vibration and finds its chief support in the analogy with the vibrations of a pendulum in a resisting medium. Bessel's investigations of this subject have, however, shown that the consideration of the actual resistance, the effect of which is chiefly to decrease the amplitude of the vibrations, is not alone sufficient to determine the motion of the pendulum. On the contrary, observation showed such a considerable increase in the period of the pendulum in consequence of damping, that the period could no longer be represented by means of the observed logarithmic decrement k. An increase in the moment of inertia

of the pendulum became necessary, which was recognized by Stokes as an effect of the internal friction of the air particles. The increase of the period expresses, therefore, the reaction of the particles of air which are set in motion by the pendulum and are carried with it. Herr Ketteler has introduced Bessel's extension of the theory of damped vibrations into optics, in his work, *Theoretische Optik gegründet auf das Bessel-Sellmeier'sche Princip.* If the equation of the damped vibration is made fundamental for the explanation of optical phenomena, Bessel's extension cannot be left out of consideration in questions concerning vibration-numbers. So long, however, as this view is used merely as a working hypothesis, it may be left an open question as to what physical conceptions are to be formed of the damping of the light-vibrations; whether, according to the older view, it is to be regarded as a decrease of the intra-molecular motion, or, according to the electro-magnetic theory, as a loss of energy which the vibrations of the valence-charges of the atoms suffer in consequence of radiation.

Taking into account this principle of Bessel, the following is to be substituted for equation (1);

$$(2) \qquad \frac{d^2x}{dt^2} + 2k_1\frac{dx}{dt} + p_1^2 x \ldots\ldots + f_1 \sin qt = 0,$$

in which the new are connected with the former coefficients by means of the equations

$$k_1 = \frac{k}{1+\gamma},\ f_1 = \frac{f}{1+\gamma},\ p_1^2 = \frac{p^2}{1+\gamma},$$

where γ is a small positive quantity.

The consequences which follow from the introduction of γ are readily seen. First, the expression now obtained for the amount of absorption, corresponding to the period $\frac{\pi}{q}$ of the incident wave, is:

$$\Delta = \frac{m f_1^2}{16 k_1^2} \sin^2 a;\ \tan a = \frac{2 k_1 q}{p_1^2 - q^2}$$

so that for $q^2 = p_1^2 = \frac{p^2}{1+\gamma}$ the absorption becomes a maximum,

which corresponds to the darkest part of the line. In consequence of this a displacement of the absorption line toward the less refrangible part of the spectrum takes place, which is measured by the difference of the periods $\frac{\pi}{p_1}$ and $\frac{\pi}{p}$. $\left(\frac{\pi}{p'} - \frac{\pi}{p} = \frac{\pi\gamma}{2p}\right)$. The expression Δ for the intensity of the absorption corresponding to q,

$$\Delta = \frac{m f_1^2}{16 k_1^2} \frac{1}{1 + \frac{(p_1^2 - q^2)^2}{4 k_1^2 q^2}}$$

shows that the line remains narrow and sharp as long as $\left(\frac{k_1}{q}\right)^2$ is small. If, therefore, γ and $\frac{k_1}{q}$ are quantities whose squares may be neglected, the damping of the vibrations essentially produces merely a displacement of the absorption lines toward the red end of the spectrum without giving rise to a noticeable broadening of the lines.

A similar consideration applies to the bright lines of the emission spectrum. For the period $\frac{t}{r_1}$ of the principal vibration emitted by the luminous gas we obtain from equation (2)

$$r_1 = \sqrt{p_1^2 - k_1^2} = \sqrt{\frac{p^2}{1+\gamma} - \left(\frac{k}{1+\gamma}\right)^2}$$

and if $\frac{k_1}{p}$ and γ are, in accordance with the previous assumption, quantities whose squares may be neglected: $\frac{\pi}{r_1} - \frac{\pi}{p} = \frac{\pi\gamma}{2p}$. The bright lines will, therefore, be displaced toward the red end of the spectrum by the same amount as the dark absorption lines. From the general expression f for the amplitude of the vibration μ:

$$f_\mu = \frac{4 k r_1}{\pi} \frac{\mu}{(k^2 + r_1^2 + \mu^2)^2 - 4 r_1^2 \mu^2}$$

or, if we place $\left(\frac{k}{p}\right)^2 = 0$

$$f_\mu = \frac{4 k p_1}{\pi} \frac{\mu}{(p_1^2 - \mu^2)^2},$$

it further follows that the ratio of the intensity of the vibration μ, lying indefinitely near $\frac{\pi}{p_1}$, to that of the principal vibration $\frac{\pi}{p_1}$,

$$\frac{I_\mu}{I_p} = \left(\frac{f_\mu}{f_p}\right)^2$$

vanishes, and that, therefore, the lines experience no perceptible broadening.

The observations of Messrs. Humphreys and Mohler are, therefore, fully explained on the assumption of a damping of the vibrations, if the quantity γ which expresses the damping may be regarded as a function of the pressure which has a definite value for any series of vibrations of the same gas, but which varies for different series and different gases. An analogy to this hypothesis is found in the theory of damped electrical vibrations, in the respect that the damping of the latter vibrations is dependent upon the form of the resonator and the dielectric constants of the medium. Taking as a first approximation for the displacement,

$$\lambda_r - \lambda_p = \frac{\gamma_0 P}{2} \lambda_r,$$

where λ_r and λ_p are the wave-lengths corresponding to the periods $\frac{\pi}{r}$ and $\frac{\pi}{p}$, and P is the pressure, we get, for $\lambda_r - \lambda_p = 2 . 10^{-8}$ mm., and $P = 12$ atmospheres, $\lambda_r = 4 . 10^{-4}$ mm., $\gamma_0 = 8 . 10^{-6}$, while $\left(\frac{k}{p}\right)^2$ is to be regarded as vanishingly small, since no broadening of the lines takes place.

The slightly damped vibrations which are emitted by the vapors of the volatile metals in the electric arc obey Kirchhoff's law, and only in the case of actual fluorescence of dissolved or solid substances, for which $\left(\frac{k}{p}\right)^2$ becomes appreciable, does a displacement of the bright bands relatively to the absorption bands take place. While metallic vapors under ordinary pressure give out only slightly damped vibrations, bright lines may be expected in their fluorescence spectrum, in addition to the broad

and diffuse bands which usually appear. Thus Messrs. E. Wiedemann and G. C. Schmidt [1] succeeded in detecting the yellow sodium lines in the fluorescence spectrum of sodium vapor, as well as fluted bands. In the case of other readily volatilized elements it appears, according to E. Wiedemann, that the presence of bright lines in the fluorescence spectrum is also possible. Of special interest would be the investigation whether these bright lines in the spectrum undergo a displacement with respect to the absorption lines, since this determination would make it possible to decide on the correctness of the assumption that $\left(\frac{k}{\rho}\right)^2 = 0$.

The investigations of Messrs. Humphreys and Mohler, the significance of which, in astrophysics, has already been pointed out by Mr. Hale,[2] deserve special interest, because, like the separation of lines observed in a magnetic field by Herr Zeemann, they demonstrate the possibility of a change in the period of vibration of the luminous molecules, while in the case of refraction and magnetic rotation of the plane of polarization there is merely a change in the velocity of propagation of the light waves. In this connection, the displacements effected by pressure have to be considered with reference to the changes in the periods of vibration, which are explained, according to Doppler's principle, by the mechanical motion of the source of light or of the observer in the line of sight. On the validity of this principle is based the method of determining the motions of the stars by measuring the displacement of lines in their spectra. If, however, displacement of the lines may also be brought about by physical causes, through definite influences to which the luminous atoms are subjected, and which may be characterized as damping of their vibrations, there arises the necessity for an investigation as to how we can separate that part of these

[1] *Sitz. d. Phys.-med. Soc. Erlangen*, November 12, 1895. This JOURNAL, 3, 207, 1896.

[2] "Note on the Application of Messrs. Jewell, Humphreys, and Mohler's Results to Certain Problems of Astrophysics." This JOURNAL, 3, 156, 1896.

changes in the periods of vibration which is due to mechanical motion from that which is due to physical causes.

The amount of the displacement of the lines which is due to the motion of the source of light, is proportional to the wave-length. The proportional factor is constant for all the lines in the spectrum of a star, and is identical with the ratio of the velocity of the star in the line of sight to the velocity of light; while the change of wave-length due to pressure is different for lines which belong to different elements and different series, although the lines may be in the same part of the spectrum. If, therefore, the metallic vapors in the absorbing layers of a star are subject to a noticeable pressure, *relative* displacements of the lines of different metals must occur, the amount of which, if the same mean pressure is assumed for all the vapors, can be made to furnish an estimate of the pressure itself. This supposition need not be regarded as contradictory to the measurements of Mr. Jewell[1] relative to the solar spectrum, since the differences in pressure, which he finds for the different metallic vapors present in the solar atmosphere, are so small that for the greater part they may be attributed to the uncertainty of the measurements. These measurements of the amount by which approximately the metallic lines in the arc-spectrum, at atmospheric pressure, are displaced with reference to the corresponding lines in the solar spectrum (0.01 to 0.02 tenth-meters), lead to the conclusion that there is a comparatively small pressure (about five atmospheres) in the Sun's reversing layer. The same conclusion very probably holds good for stars of the second type, whose physical condition evidently closely resembles that of the Sun. For these stars the displacement due to pressure, amounting to about 0.02 tenth-meters (which, according to Doppler's principle, corresponds to a velocity of 1.5 kilometers at λ 4000) will therefore hardly reach the limit of accuracy of Herr Vogel's[2] measurements of the motion of the stars in the line of sight.

[1] "Note on the Pressure of the Reversing Layer of the Solar Atmosphere." This JOURNAL, 3, 138, 1896.

[2] *Pub. Obs. Potsdam*, 7, 25.

In the case of stars of the first type, whose spectra contain but few and weak metallic lines, the displacements of the artificially produced hydrogen lines, with reference to the hydrogen lines in the star spectra, will in most cases have to be measured directly. The investigations of Messrs. Humphreys and Mohler do not include hydrogen; nor can it be assumed that the pressure of the gas has alone any considerable influence on the period of vibration, since, under increased pressure, its lines broaden so greatly that even at a pressure of less than one atmosphere the spectrum becomes almost continuous. The significance of the displacement of the hydrogen lines in a stellar spectrum, in accordance with Doppler's principle is, therefore, not in the least doubtful. Of special significance for investigations of stars of Type I are Herr Vogel's[1] measurements in the spectrum of Sirius, from which no appreciable displacement of the absorption lines, due to pressure, can be deduced. The measurements of the displacements of the $H\gamma$ line in the stellar spectrum, with respect to the corresponding line produced by means of Geissler tubes, gave the following values:

1891, March 21 — 14.9 kilometers (first plate),
March 21 — 15.9 kilometers (second plate),
March 22 — 15.0 kilometers,

15.2 kilometers,

while the determinations of the displacement of the fine iron lines in the star spectrum, with respect to the corresponding lines of the iron spectrum, photographed on the same plate, furnished the values:

1891, March 21 — 13.4 kilometers,
March 21 — 14.9 kilometers,
March 22 — 16.3 kilometers,

14.9 kilometers.

The slight difference between these numbers is, considering the peculiar difficulty of such measurements, to be ascribed

[1] *Pub. Obs. Potsdam*, 7, 75 and 82.

solely to physiological causes, so that there is no displacement of the metallic lines with respect to the hydrogen lines.

According to these views, an effect of the differences in pressure on the position of the dark lines in the spectrum is to be expected only at the present limit of precision for the most accurate determinations of wave-lengths in stellar spectra. Therefore, in the determination of the motion of the stars by Doppler's principle, the wave-length may still be regarded as constant.

This statement cannot be made with equal certainty in the case of the emission spectrum. Since the number of stars whose spectra contain bright lines is comparatively very small, it is evident, on this ground alone, that the physical conditions under which the vapors on the surface of these stars are luminous, is to be regarded as anomalous.

This is especially true with reference to new stars. The occurrence of pairs of lines seems to be characteristic of their spectrum, which arises from the superposition and relative displacement of the emission spectrum and the absorption spectrum of the gases. Here the bright lines lie on the less refrangible side of the dark lines, precisely as in the case of the bright bands of fluorescing substances. The agreement in the direction of these displacements for different stars, and their surprising amount, oppose considerable difficulties to the purely mechanical explanation based on the motion of one or more luminous bodies, and appear to favor more directly the assumption of an essentially physical cause.

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CORRESPONDENCE OF THE PHOTOGRAPHIC DURCHMUSTERUNG WITH THE VISUAL.

By R. H. TUCKER.

THE first volume of the *Cape Photographic Durchmusterung* has been published, and it forms the most extensive contribution to the needs of the practical observer that the photographic process has furnished.

In considering the results as a basis for present use, and for further development, the true aim of the *Durchmusterung* must be kept plainly in view. It is, briefly, to give a census of the stars, which shall be complete down to a certain definite limit of brightness. To furnish positions, within such limits of error as shall make it possible to easily identify the stars, and to set for them for purposes of accurate determination. And, finally, to rank them on a scale of magnitudes, which shall be in accord with certain standards, and shall be in true proportion throughout. If these several purposes be set down according to their importance, the order would be: completeness, uniformity of scale, and exactness of position; the last ranking considerably below the other two. Each purpose is liable to fall short of perfection in attainment, from a variety of causes; no work is free from the evidence of our own fallibility. There may be expected a certain number of mistakes, as long as the human observer has aught to do with investigation, even if his part should, sometime, be reduced to the transcription only, of results. In this class of work, there will be also systematic and accidental errors, affecting both the scale of magnitudes and the exactness of the places. Whatever limits of error be adopted, there are more to be feared the chance combinations and mistakes, that will occasionally produce abnormal results. The scope of the *Cordoba Visual Durchmusterung* was designed to be the tenth magnitude, to which limit the enumeration was intended to be complete. With respect to the scope of the *Photographic*,

Dr. Gill states, on page xxiii of the Introduction, that "probably every catalogued plate shows every star which Argelander would have considered it essential to include in a *Durchmusterung* similar to his own, with the exception, perhaps, of a very few red stars of the fainter class."

The ninth magnitude of the visual scale would thus be the limit of the complete census contemplated; while, necessarily, in order to be sure of including that grade, fainter stars will be gathered in, to a considerable extent.

Internal evidence, of some weight, may always be derived from a work of this character, as to its successful fulfillment of purpose; besides the authority, which is given by the ability and repute of the designers and constructors. Some such evidence is furnished by the count of the grades of the scale, to be given, later, in the tables.

The discussion of the scale itself has been made in the second part of the Introduction, by Professor Kapteyn, with comprehensive thoroughness. The magnitudes of the *Cordoba Zone Catalogue* have been adopted as the standard, the reduction is differential, and the aim has been to produce, from the photographic plates, a scale of magnitudes which shall be the same as the visual scale.

The results of an earlier investigation, by the same author, upon the distribution and brightness of the stars, as shown by photography, are quoted. In discussing the star density, while the comparison with Schönfelds *DM.* gives a legitimate conclusion, I do not think that the same validity attaches to that made with Gould's Zones.

The aim of the former was a complete enumeration of the stars. The aim of the zone work, on the contrary, was to obtain a large number of stars, generally distributed. The following concise statement of Dr. Gould's, which I quote from a letter written in 1893, makes this quite plain: "The object in view was to get as many stars as possible — not to make an exhaustive catalogue down to any limit of magnitude."

With respect to the light of the stars, Kapteyn's conclusions

are, in brief, that the effect of stars of the same visual magnitude, upon the photographic plate, varies in different parts of the sky. And that "the stars of the Milky Way are in general bluer than the stars in other regions of the sky." Thus far, from his earlier investigation. Resulting from the detailed comparisons, in the Introduction, the conclusion is drawn, also, that "the uniformity of color for the fainter stars is apparently much greater than that for the brighter stars."

In his comparison of the *Photographic DM.* and the *Visual*, a marked difference is found between the respective scales, depending apparently upon the galactic latitude. The photographic scale is intended to reproduce that of the *Zone Catalogue*. Accordingly, if the magnitudes of the visual *DM.* be assumed to be correct, "then the fainter stars of the *Z. C.* have been estimated half a magnitude *brighter* in the Milky Way than at the poles of that belt."

It is possible that the scale of either may be subject to systematic variation. The advantage would seem to lie with the zone work, for conditions favorable to consistency of scale, were it not for the fact that the consistent estimation of magnitudes was recognized as so important a part of the other work; while in the zones it was subordinate to the exact and rapid determinations of position. The effect of a bright background, due to the accumulated mass of stars in the Milky Way, would naturally reduce the apparent brilliancy of individual stars, observed in an unilluminated field, for the *Durchmusterung*. For the *Zone Catalogue*, the artificial illumination, required for meridian circle observations, would be so much in excess of the sky illumination, that the effect of the latter would be imperceptible.

Nevertheless, I am inclined to believe that it was possible for the *DM.* observers, with the state of the sky visibly changing, as the Milky Way was entered, to carry the same scale into and through those dense regions, without sensible variation. The general conditions were good, and were, as far as possible, uniform. Certainly a great amount of practice was obtained, under constant exercise of the same faculties. My own experi-

ence included three-fifths of the observations, between —22° and —42°, and exactly half the revision, devoted to the settlement of doubtful cases.

The scale of the visual *DM.* was recognized, during the progress of reduction, to have a fairly consistent difference from that of the *Zone Catalogue*. I derived all the constants of reduction for the twenty degrees, comparing every hour of observation with all the stars of the catalogue. This was, ordinarily, accomplished within the month following the observations; though a few stars, for the determination of the exact limits of the zone observed, were compared on the day immediately following.

This difference may be stated, in general, to have increased gradually, up to one quarter of a magnitude at about 9; so that our 9 corresponded best with a type between 9 and 9½ of the *Zone Catalogue*, the individual estimates of which were made originally to half magnitudes. The 10 of the *Zone Catalogue* was usually found to be brighter than the 9½, and it appeared probable that the few observations made of that estimate were of brighter stars, under unfavorable conditions.

On the other hand, considering the photographic work by itself, the effect of a bright background, upon the plates, should be taken into account. This might be similar to the effect of moonlight, or of previous exposure to faint light, referred to by Dr. Gill, p. xxiii of the Introduction. There seems to be need of much further investigation of such features of the photographic process. And it is suggested by the many interesting points, developed in the treatment of the results, in part second.

That the scale of the photographic *DM.* bears quite a different relation to that of the visual, for the Milky Way and for regions outside, will be clearly evident in the comparisons which will follow.

The photographic scale has been compared by Professor Kapteyn, for the part common to Schönfeld's *DM.;* and with the *Cordoba DM.* for portions of the belt —22° to —32°, about 1300 stars having been used in the last. The Cordoba volume,

for the ten degrees next south, was not available at that time for further comparison. This has suggested an extension to the field covered by the second volume of the visual *Durchmusterung*.

The first point that attracts attention is the evident break in the continuity of the photographic enumeration, that occurs at $-34°$. This corresponds with the employment of the repolished object-glass for the photographs; and, apparently, with the use of plates of a different manufacture. The number of stars per degree rises from an average of 6700 on one side, to 9500 on the other. If these figures be translated into the equivalent of a zone one degree wide at the equator, the average rises from about 8000 to nearly 12,000. If there were any difference between the adjoining degrees, with relation to the Milky Way, an explanation might thus be given for the change. But the zones, one degree in width, cross the branches of the Milky Way quite symmetrically, with respect to each other.

Turning now to the visual *DM.* there is no break in the uniformity of the enumeration at this point. The average equivalent, for an equatorial zone of one degree, is 20,200 for the whole extent from $-22°$ to $-42°$; and the average deviation is 4 per cent., while it does not exceed 14 per cent. for any degree.

In other terms, the photographic enumeration increases from an average of 42 per cent. of the visual for the first twelve degrees common to both, up to 59 per cent. for the four degrees next south of $-34°$.

For these comparisons a belt from 20′ to 30′ of declination has accordingly been chosen from each of the two degrees, -33 and -34. Each belt extends through the complete circuit of twenty-four hours, and above 2500 stars are included, nearly double the number used by Professor Kapteyn for the northern portion. Each belt has been divided into parts, in order to represent the Milky Way separately. It is, perhaps, needless to say that in identifying the stars, a scrutiny is made of the minutes of declination, adjoining the exact limits of the zone chosen. The stars of the Uranometry have not been included.

The magnitudes of the visual *DM.* for these stars were taken from the Uranometry, although the brightness was always estimated at the time of observation. In the second part of the photographic Introduction, Professor Kapteyn calls attention to a possible difference in the relation between telescopic and naked eye estimates, as affecting the photographic adjustment required for the plates. This is one of the many interesting points that will repay investigation by those who are following the development of astronomical photography. As illustrating one aim of these comparisons, the testing of the general consistency of the scale, there is given here in Table I the results of a count of the magnitudes of the visual scale, by tenths, which I made while in Cordoba, and which has not before been published in its entirety. It embraces the first ten degrees, from 0^h to 5^h inclusive; and the first five degrees for 7^h, this last representing the dense part of the Milky Way. Nearly 37,000 stars were counted. A small portion of these results (2000 stars in the first five degrees of 0^h), forms a part of the *DM.* Introduction, and had been previously printed in the *Astronomical Journal*, No. 230.

The individual tenths are fairly represented, as shown by the third column of percentages. The summation is given in the fourth column. By inspection of the third column it will be seen that the 10 includes a percentage equivalent to a total enumeration to 10¼ inclusive, on the same scale as that of the stars above it.

On the hypothesis of distribution by magnitudes, which has been found to represent the northern hemisphere, down to 9 mag. there would be 323,000 stars in the area $-22°$ to $-42°$, down to 10¾ inclusive. The visual *DM.* actually has 340,000 in this area. There are two hypotheses to be adopted, however, in making this comparison; the same distribution for the two hemispheres, and the validity of the law beyond 9 mag.

As illustrating the possible range of the visual observations, for the telescope used, it served for the identification of an asteroid noted as twelfth magnitude, upon one occasion. It had

TABLE I.

CORDOBA VISUAL DM.

Mag.	Stars	Per 1000	Sum
To 6.9	202	6	6
7.0	66	2	8
7.1	26	1	9
7.2	53	1	10
7.3	54	1	11
7.4	28	1	12
7.5	97	3	15
7.6	54	1	16
7.7	104	3	19
7.8	78	2	21
7.9	57	2	23
8.0	184	5	28
8.1	137	4	32
8.2	183	5	37
8.3	217	6	43
8.4	159	4	47
8.5	366	10	57
8.6	300	8	65
8.7	421	11	76
8.8	370	10	86
8.9	400	11	97
9.0	871	24	121
9.1	852	23	144
9.2	959	26	170
9.3	978	27	197
9.4	1220	33	230
9.5	2000	54	284
9.6	2305	63	347
9.7	3216	87	434
9.8	3704	100	534
9.9	3171	86	620
10.0	13966	380	1000
Total	36798	1000	

been the custom to observe some of the asteroids that came to opposition far south, and of which determinations were desired. For this particular one, the ephemeris place was so uncertain that repeated trials failed to identify it in the 13-inch telescope by reason of the large number of faint stars within the area. I proposed trying for it with the *DM.* telescope. Its

aperture, five inches, should have rendered a 12 mag. just barely visible. I swept over the region in the usual manner of observing for the *DM.*, taking only the faintest stars, and a few brighter ones for identification. The following night a similar sweep was made and the asteroid was at once picked out from the reduction sheets with a difference in place corresponding to the daily motion. I have always regretted not having again turned to the region to see how the asteroid compared in light with the faintest stars visible.

In Table I, the constant $b=3.9$, in the expression $a\,b^m$ for the sum of the stars down to any magnitude is clearly confirmed for the grades 7, 8 and 9. If, however, we look for the place of 10.0 by extension of the same hypothesis, it would be found at 9¾ of the scale, and there would thus be an indication that the stars estimated fainter than that really belong to 10 mag. From the manner in which the catalogue magnitudes are derived, the direct arithmetical mean of the separate estimates, the 9.8 and 9.9 were all observed once at least as 10, and it is not improbable that these stars may be truly tenth magnitude. The scale would then include a number of stars equivalent to a complete enumeration to 10½ inclusive. This is confirmed again by finding the percentage which is one-fourth of the total, at 9½ of the scale.

It must be borne in mind here and in what follows that the 10 of the visual scale is a flexible grade, including all stars estimated as tenth magnitude and fainter. It was in no sense our object to record the faintest stars visible, but to include all of which there was any doubt with respect to being approximately near the limit. In taking the means, it is always reckoned as 10.0, although the actual estimation may have been, and undoubtedly often was, consciously, beyond.

Referring again to the number of stars included in the photographic *DM.*, the total for 19° would indicate an equivalent of the complete enumeration down to 10¼ nearly, according to the law of distribution for the northern hemisphere. This limit would be different for the two parts above and below

$-34°$; the law giving 10.1 and 10.5, respectively, for the areas each side.

In terms of the visual scale, 42 per cent. would fall at 9.7, and 59 per cent. would reach nearly to 9.9, or, in round numbers, the limit has been extended one-fourth of a magnitude, after passing $-34°$. The actual scope of the work, the limit of brightness to which it is complete, will be much above. The count of scale in Table IV, and the number of stars missing from the fainter grades, Table V, must be referred to, for evidence from within and without, as to the completeness.

With respect to the law of distribution, results greatly at variance with the accepted constants have been found in the course of charting faint stars for standards of magnitude with the 36-inch telescope of this Observatory. A series of twelve small charts were completed, in extension of the scheme, originally of Professor Pickering (*M. N.*, **45**, 124), to furnish standards for comparison. The stars were grouped here in five classes for convenience in charting: the first including all to 9 mag. and each class below extending two magnitudes. The extent of area covered is too small—two square degrees—to base any definite conclusions upon, but the count indicates an entire change in the constants used in summing up magnitudes brighter than 9. The results are given here in condensed form:

Mag.	Stars	Sum	Per cent.
To 9	16	16	.011
10 and 11	91	107	.076
12 and 13	192	299	.214
14 and 15	327	626	.447
16 and 17	770	1396	1.000

The b of the formula, computed by extending the summation of 9 mag. to 17 mag., would be 1.75; in place of 3.91, adopted up to 9 inclusive.

Taking up in order the results of the *DM.* comparison for $-33°$, Table II has been formed by adopting, first, the visual magnitudes as the standard and grouping the individual stars as follows:

The magnitudes 7 to 8.
The magnitude 8.0 alone.
The magnitudes 8.1, 8.2, 8.3 together.
The magnitudes 8.4, 8.5, 8.6 together.
The magnitudes 8.7, 8.8, 8.9 together.

And so on up to 10. This distribution by groups is somewhat an arbitrary one, since perhaps the 8.9 belongs more precisely to 9 as the nearest quarter. But as there is an acknowledged tendency to gravitate towards the even magnitudes in direct observation, the whole units have been given a separate class. The table shows that a fair distribution of both scales has been effected, but Table IV should be consulted for final illustration of this point.

There are tendencies, also, in the photographic scale to exclude certain tenths; and there is great divergence both as to the limits and the tenths chosen in different portions. A few illustrations may be given. In $8^h - 34°$, for the zone compared, there are omitted entirely 9.5, 9.7, 9.9, 10.1, 10.3 and 10.5, while the scale includes 10.7 and 10.8; and the tenths omitted here are recorded commonly for other portions. The recorded limits vary from 9.6 to 11.4 over stretches of considerable length. In the zone through $5^h - 34°$, 60 per cent. of the stars are 10 mag. or fainter, almost to the exclusion of faint estimates between 9 and 10. While in 6^h, next following, but 20 per cent. are as faint as 10, and those mainly confined to the first half hour. These variations are, however, to be clearly anticipated, since by means of the photographic process the scale, founded upon only the brightest stars, is to be extended down. Thus the exposure, the sensitiveness of particular plates, and other conditions, must be taken into account. These difficulties are referred to by Dr. Gill, and they are most extensively treated in Scheiner's recent book upon Celestial Photography. The same class of uncertainty arises whenever differential work has to be extended beyond the base upon which it rests.

In making up the means of Table II, the visual 10 has been reckoned at 10.0. The number of stars in each group is not a

test of the distribution by quarters, since there are not included the stars of either catalogue, omitted by the other. For the photographic *DM.*, these begin to mount up perceptibly after passing 9 mag.

TABLE II.

COMPARISON OF VISUAL AND PHOTOGRAPHIC MAGNITUDES.—33°.

Visual standard				Photographic standard				Mean difference		
Vis.	Phot.	Stars	V.-P.	Phot.	Vis.	Stars	V.-P.	Mag.	V.-P.	Wt.
7.52	7.88	47	—.36	7.62	7.52	30	— .10	7½	—.26	4
8.0	8.34	21	—.34	8.0	8.04	14	+ .04	8	—.19	2
8.23	8.40	23	—.17	8.22	8.10	36	— .12	8¼	—.14	3
8.50	8.74	68	—.24	8.51	8.56	68	+ .05	8½	—.09	7
8.80	9.01	74	—.21	8.81	8.92	61	+ .11	8¾	—.06	7
9.0	9.15	77	—.15	9.0	9.05	59	+ .05	9	—.06	7
9.25	9.48	132	—.23	9.19	9.20	97	+ .01	9¼	—.13	11
9.52	9.70	241	—.18	9.51	9.42	216	— .09	9½	—.14	23
9.79	9.90	226	—.11	9.80	9.64	162	— .16	9¾	—.13	19
10.	10.08	111	—.08	10.0	9.70	91	— .30	10	—.18	10
				10.20	9.77	110	(— .43)			
		1020		10.45	9.73	58	(— .72)	Mean	—.14	
				10.80	9.75	12	(—1.05)			
				11.0	9.90	6	(—1.10)			

Seventeen stars from the *Uranometria Argentina* brighter than 7 have been omitted. Also eight differences of magnitude:

Vis.	Phot.
8.7	10.2
8.9	10.2
dup.	(2)
8.7	10.2
8.3	9.8
8.7	10.2
8.7	10.0
9.5	11.4

The same process is repeated for the second part of the table, except that the stars are combined with the photographic estimates as the standard. Thus each star is represented by a double comparison. The differences are taken out in all cases in the sense *Visual—Photographic*, and they thus conform to the

comparisons made by Professor Kapteyn. In the final tabulation of the Mean Difference, the results of the two preceding sections have been combined strictly according to the number of stars. The magnitudes may, then, be considered as representing types midway between those of the two scales; but the mean results are but slightly affected, by leaving out of account the systematic differences, in the combination. The scale of one section should be moved along upon that of the other in order to allow for systematic difference, and the magnitudes could thus be tabulated for the scale of either standard. The 10.0 of the photographic scale has been combined directly with the 10 of the visual for the final section, without carrying forward the fainter mean types of the photographic. The individual estimates have, however, been included in the first section with the visual standard. The last column of weights is made up with a basis of ten double comparisons as the unit.

The comparisons by the two standards show mutual progression of the two scales up to 10.0 of the photographic. From 10 to 11, however, there is apparently represented nearly the same type of the visual. But there is no way of distinguishing a star visually fainter than 10; and it is evident that if a group be taken, including any number of tens, the mean of the group will be brighter than 10 if one or more stars were estimated brighter. The 10 of the visual scale is probably best defined by the result in the first section alone.

The mean value for this table is, $V.-P.=-0.14$ mag., whether taken directly or with weights for the separate classes, and there is not much deviation from uniformity. The photographic estimates are fainter than the visual.

The explanation which has been given of Table II will apply, wholly, or in part, to the tables that follow. The method of combination, for the discussion of differences of magnitude, is the same that I used in all of the earlier comparisons of the scale with various catalogues, the results of which will be found in the Introduction to the *Cordoba DM.*

In order to represent the Milky Way separately it will not

be necessary to give the detailed results, in the full form of the table preceding. Accordingly there is condensed, in Table III, the final or mean differences, derived precisely in the same manner as those of the last section of Table II for the complete belt. The third and fifth columns include the Milky Way, and in each column is given the number, half the sum of the comparisons by both standards. The total for the column will be less than the whole number of stars compared, by one half the photographic estimates fainter than the 10.0.

The scale of the photographic *DM.* is seen to be in general one quarter fainter than the visual, outside of the Milky Way: while within those regions occur the only plus signs, indicating that the photographic is brighter than the visual. The change is more noticeable in the first crossing of the Milky Way.

TABLE III.

V.–P. GROUPS OF R. A. —33°

Mag.	1h–5h	No.	6h–8h	No.	9h–15h	No.	16h–18h	No.	19h–23h	No.
7½	—.10	7	—.40	4	—.37	12	—.28	9	—.10	6
8	—.15	3	—.26	2	—.29	4	—.20	2	—.09	5
8¼	—.20	7	—.11	4	—.17	10	+.04	3	—.10	4
8½	—.06	16	—.01	12	—.22	21	+.12	7	—.14	11
8¾	—.04	11	+.13	21	—.32	19	+.03	7	—.02	8
9	—.08	12	+.22	17	—.32	14	—.07	11	—.15	13
9¼	—.07	22	+.13	21	—.28	37	—.01	15	—.29	18
9½	—.28	40	+.16	57	—.30	52	—.08	44	—.28	34
9¾	—.31	24	+.15	54	—.20	29	—.17	64	—.43	22
10	—.51	9	+.03	46	—.33	5	—.23	31	—.38	14
Stars	173		246		219		229		153	

The dense parts of the Milky Way are: 7^h–8^h; and 17^h–18^h.

In Table IV, following, there is given the count of both catalogues for the zone compared, including in each, such stars as have been omitted from the other. The number of stars is given for the separate quarters, and the summation by quarters, with the percentages represented by these numbers.

The count of the visual scale closely resembles that of Table I, although the total includes but 7 per cent. of the number of stars. In comparing, the limiting tenths of the quarters must be reckoned: 8.3, 8.6, and 8.9 for 8¼, 8½, and 8¾. The agreement of the scale at 6.9, 7.9, and 8.9 with the constant *b* of the law of distribution, is excellent; but the tenth magnitude would evidently fall at 9.8 of the scale, by extending with the same constant.

The photographic scale, in the brighter part, does not conform to the theory, the stars from 7 to 8 falling behind. But this is possibly due to fainter estimates for the *U. A.* stars, which estimates have not been counted. The sums at 8¼ and 9¼ are in accord, and if all the faintest stars be grouped under magnitude ten, the 9 and 10 would agree with theory. In fact, the faint estimates are in such a small proportion to the stars above them, that one would be inclined to classify them farther up the scale, if uniformity were assumed. Considering the small variation of color, accepted for the fainter stars, there should be no reason for stars of 10¾ being photographed, when those of 9¾ do not produce an impression. And there is a noticeable falling off at 9½ of the scale, considered by itself entirely, which is ample internal evidence of the approach to the limit. It would appear that the expressions, by means of which the curves for magnitudes were extended below the standards, are not entirely in accord with the generally accepted increase in the distribution of the fainter stars. Of course a number of different plates are represented in this zone, but the characteristics of the whole belt are noticeable, as well, for smaller positions. In the dense parts of the Milky Way, the lower grades are, proportionally, much better represented.

As regards the scope of the photographic *DM.* the sums at 9 and 9½ would indicate, by the law of progression, that enough stars had been included up to this last grade. But by comparing the two catalogues the number at 9½ of the photographic is found to be not up to the scale of the visual, even allowing the proportional number of stars for the systematic

difference in scale. If, however, we take the sum at 9 mag. and make the proper allowance for the systematic difference (Table II) we find it covers the number at 9 visual, and the photographic *DM.* would appear to be complete to ninth magnitude of its own scale. Since the visual includes every star, estimated by the photographic as bright as 9.0, the completeness of the former at that point is pretty well assured.

TABLE IV.

COUNT AND PERCENTAGE. -33° 20′ TO 30′.

Visual DM.					Photographic DM.				
Mag.	Stars	Sum.	Per 1000	Sum.	Mag.	Stars	Sum.	Per 1000	Sum.
U. A.	17	17	6	6	U. A.	17	17	16	16
7–8	47	64	17	23	7–8	30	47	28	44
8	21	185	7	30	8	14	61	13	57
8¼	24	109	8	38	8¼	36	97	34	91
8½	68	277	25	63	8½	68	165	63	154
8¾	81	58	29	92	8¾	61	226	57	211
9	80	338	29	121	9	59	285	55	266
9¼	161	499	58	179	9¼	98	383	91	357
9½	395	894	143	322	9½	218	601	202	559
9¾	714	1608	260	582	9¾	169	770	157	716
10	1149	2757	418	1000	10	101	871	94	810
					10¼	121	992	112	922
					10½	64	1056	60	982
					10¾	12	1068	11	993
					11	7	1075	7	1000

The Uranometry stars include all to 6.9 mag.

There follows in Table V a list of the number of stars missing from either catalogue. It seems evident that the tenth magnitude has been generally included in the visual work. The missing stars occur entirely in the Milky Way, where the difficulty of observing all the stars in the densest parts would account for an occasional omission. For more than eighteen hours of the twenty-four the visual includes every star given by the photographic in this zone. Many of the stars missing

belong to pairs, or triple stars, for the separate components of which it would always be difficult to make distinct records. The photographic method presents here a decided advantage, in that no greater speed is required in measuring the most closely packed regions, and pairs can be picked apart at leisure. The rapidity of observation was not necessarily productive of bad estimates of magnitude, however; one's faculties are trained to alertness and to decision of judgment in such work, and it does not deteriorate with speed.

The stars missing from the photographic *DM.* are mainly fainter than 9 of the visual scale. The few exceptions may, very probably, be of stars estimated too bright in the visual work. All brighter than 9.2 lie outside of the Milky Way; and in general the proportion of missing stars is decidedly smaller in the Milky Way than in regions outside, corresponding with the evident difference in the relation of the two scales.

Of the visual 10 that have been included in the photographic work three-quarters for this zone are within the Milky Way.

TABLE V.

STARS MISSING. —33° 20′ TO 30′.

From Vis. DM.[1]			From Phot. DM.		
No.	Phot. Mag.		No.	Vis. Mag.	Sum.
1	9.1	This star is close to 4 mag.	1	8.7	1
2	9.4	One of these is close to 7 mag.	1	8.9	2
3	9.7	One belongs to very close pair.	3	9.0	5
2	9.8	Both belong to wide pairs.	7	9.1	12
1	9.9	Close to 7 mag.	2	9.2	14
21	10.0 to 10.5	Some of these belong to pairs.	20	9.3	34
—			22	9.4	56
30	Total	All of these stars are in the Milky Way, 5^h to 8^h; and 17^h to 19^h.	40	9.5	96
			90	9.6	186
			118	9.7	304
			98	9.8	402
			272	9.9	674
			1038	10.	1712

[1] I have looked up two cases of the brighter stars missing from the visual *DM.* with the meridian circle here. They were each of tenth magnitude.

The accidental error of the magnitudes deserves some consideration. The average difference between the respective estimates for the same star, and without taking account of the systematic difference between the two scales, is ±0.30, including all between 7 and 10 of the photographic scale. The stars fainter than 10.0 of this last have not been included, in the mean; they appear to belong to the grade of faint ten.

The 10 of the visual has been rated at 10.0, and since but 10 per cent. of the visual tens are included in the photographic in this zone it seems probable that those included belong to the brightest class, near an actual 10 of the visual scale. The average difference does not show much variation between different parts of the scale, but is sensibly smaller for the faint stars from 9½ to 9¾.

The probable error of a magnitude, for stars of 10 or brighter, taken from either catalogue, would be ±0.2 assigning the same weight to both authorities.

At the same time that these comparisons of magnitude were made, the difference between the respective places has been taken out. The average difference in right ascension is ±0ˢ.90. There is a considerable increase within the Milky Way, where it slightly exceeds 1ˢ. No account has been made of systematic difference, but there is a decided tendency towards *smaller* right ascensions in the faint stars of the visual. This is unquestionably due to the fact that the stars were generally recorded at the time of extinction behind the opaque bar in the field of view. The faint stars disappeared at contact, nearly, while the brighter ones required a perceptible interval to pass from sight. The brighter stars, from which the reduction constants were necessarily derived, would thus be recorded too late with respect to the others. This difficulty was recognized at the time, and it was intended to record the *bisection* of the bright stars by the edge of the bar. In the most hurried portions there was often not time to watch individual stars steadily, to obtain real bisections, and it is in these portions that the systematic difference is most marked. It amounts to one second,

evidently, for considerable stretches of the Milky Way, the faint stars requiring a *plus* correction to the right ascension.

There are often a number of stars coming to transit within one second. In observing such combinations two or more were recorded for the same tap of the chronograph key; but confusion between observer and recorder, or in the observer's recollection of the group, was apt to result from combining more than two. It was accordingly necessary, at times, to tap consciously too early for some stars, on the approach of a group, in order not to have the taps too close on the chronograph, and in order to get in the necessary calls for the recorder without confusion. A star had often to be recorded when it had fairly passed contact, by including it in the record for the last preceding star. Thus there enter exceptional sources of occasional large errors in the visual right ascensions, especially in dense groups of stars. Less than 1 per cent. of the differences have been excluded in making this comparison, which may be taken to fairly represent the limit of accidental error. The *probable errors* to be derived from the material, adjusting the respective errors in accord with the weight derived from direct comparison with catalogue places, would be

p. e. Phot. $\pm 0^s.4$
p. e. Visual $\pm 0^s.7$

With respect to declinations the speed required in the denser regions had, in general, much less injurious influence; it was simply a matter of accustoming oneself to rapid and decisive estimates of tenths of the divided scale. The average difference of declination is $\pm 0'.34$, slightly larger in the Milky Way than outside, no systematic error being evident, and without rejecting any discordant differences. As *probable error* this would be distributed between the two catalogues:

p. e. Phot. $\pm 0'.06$
p. e. Visual $\pm 0'.28$

It must be borne in mind that the unit of measure in the visual work is one minute of arc. The scale was divided into $10'$ spaces, and estimated to tenths only. It is evident that if

no errors were made in the observation, and the nearest tenth *invariably* recorded, the average error of a single determination of declination would be one-quarter of a minute, and one in five would have an error of half a minute.

It is not likely that any observer records the nearest tenth invariably. There is often a distinct personality, by which certain tenths are more often chosen. Thus between 2 and 3 and between 7 and 8 the uncertainty can well become real. I made tests, by counts of the tenths estimated by myself, at various times. The only noticeable tendency was to avoid the 5. As the point exactly half way is certainly 5, I apparently estimated the slightest shade either side of that, as 4 or 6. The declinations of the photographic work have been measured with the tenth of a minute as the unit. In considering the results of a comparison between the two systems, this should not be lost sight of; the absolute uncertainty of a determination, aside from the errors of observation, or of measurement, is ten times as large in one case as in the other.

For the declinations of the *Bonn Southern DM.*, Schönfeld used a scale divided into spaces $4'.5$ long. His unit of measure, consequently, comes between the other two. This point has nowhere been in evidence, in the comparisons which I have seen.

The probable errors, derived from comparisons of the brighter stars with catalogue places, are given in the Introduction to the Cordoba *DM.* All of those comparisons were made by me, and I see no need of further extending or of modifying them. But the accidental error of observation is best derived from the comparison of the individual determinations, and it is no larger for faint stars than for the brighter ones.

From a large number of zones, discussed at various stages of the work, and from the compilation of the separate determinations, the following values represent the average probable error of one observation :

p. e. R. A. $\pm 0^s.7$.
p. e. Dec. $\pm 0'.3$.

In the catalogue, the comparison with the photographic shows that differences between the right ascensions of both, amounting to 3^s, are not uncommon, including the systematic difference. In declination the maximum difference is about $2'$, and it rarely exceeds $1'$. These errors, and their limits are matters of importance to those using the catalogues; there is often an evident lack of judgment with respect to the positions. The uncertainty may be safely stated as not exceeding, on the average, the respective units: one quarter of a magnitude in brightness, one second of time in right ascension, and one minute of arc in declination. These may be fairly said to conform to the purpose of the work, and further refinement is not essential.

The results for the $10'$ zone $-34°$ can be briefly summed

TABLE VI.

COMPARISON OF VISUAL AND PHOTOGRAPHIC MAGNITUDES. $-34°$.

Visual standard				Photographic standard				Mean difference		
Vis.	Phot.	Stars	V.−P.	Phot.	Vis.	Stars	V.−P.	Mag.	V.−P.	Wt.
7.51	7.88	47	−.37	7.56	7.58	32	+.02	7½	−.21	4
8.0	8.27	21	−.27	8.0	8.04	9	+.04	8	−.17	1
8.27	8.54	35	−.27	8.20	8.01	30	−.19	8¼	−.23	3
8.51	8.79	65	−.28	8.51	8.52	72	+.01	8½	−.13	7
8.78	9.06	77	−.28	8.81	8.93	75	+.12	8¾	−.08	8
9.0	9.24	77	−.24	9.0	9.01	59	+.01	9	−.13	7
9.24	9.41	145	−.17	9.20	9.31	145	+.11	9¼	−.03	14
9.53	9.69	323	−.16	9.51	9.46	275	−.05	9½	−.11	30
9.80	9.90	411	−.10	9.79	9.64	248	−.15	9¾	−.12	33
10	10.15	295	−.15	10.0	9.74	202	−.26	10	−.19	25
				10.19	9.84	176	(−.35)			
		1496		10.47	9.87	126	(−.60)			
				10.78	9.91	47	(−.87)	Mean	−.14	

17 stars brighter than 7, from the *Uranometria Argentina*, have been omitted. Also 6 differences of magnitude.

Vis.	Phot.	Vis.	Phot.
9.3	10.8	dup.	(2)
9.4	10.8	8.2	9.4
8.8	10.2	8.0	9.4

up, they have been derived in the same manner precisely as for the preceding degree. Table VI includes the comparison of magnitudes for the whole belt. The mean difference is —0.14, or by weight —0.125, the photographic scale being brighter throughout. The differences, with the visual estimates as standard, indicate a more nearly uniform progression in the types. The final mean difference should have largely eliminated variations due to individual estimates.

The average difference of magnitude for the two scales from 7 to 10 of the photographic, without taking account of systematic difference, is ±0.29. This varies, however, from ±0.18 in one branch of the Milky Way, to ±0.40 in the other; the values for stars outside being in accord with the mean. The average difference in right ascension is ±0ˢ.99, ranging from ±0.93 outside the Milky Way to ±1ˢ.11 within, where the systematic error for the faint stars is most marked. The mean difference would be much decreased by application of the systematic correction. The average difference in declination is ±0′.33. These differ but slightly from the preceding degree, and no further discussion appears necessary.

TABLE VII.

V.—P. GROUPS OF R. A. —34°.

Mag.[1]	0h–6h	No.	7h–8h	No.	9h–11h	No.
7½	—.01	8	.....	..	—.32	9
8	+.04	5	.00	1	+.10	1
8¼	+.01	8	—.11	3	—.41	3
8½	—.07	17	+.15	4	—.32	9
8¾	—.06	15	—.11	7	—.45	8
9	—.38	11	+.03	10	—.24	10
9¼	—.20	22	—.01	17	—.15	14
9½	—.27	39	+.02	36	—.16	51
9¾	—.28	38	—.03	66	—.27	62
10	—.44	14	—.19	90	—.30	57
Stars	200	..	290	..	288	..

[1] The Milky Way is 7h to 8h; and 17h to 18h.

TABLE VII—*Continued.*

V.–P. GROUPS OF R. A. —34°.

Mag.[1]	12h–16h	No.	17h–18h	No.	19h–23h	No.
7½	—.45	9	—.06	6	—.13	6
8	—.43	3	+.10	1	—.31	2
8¼	—.41	7	—.26	2	—.27	8
8½	—.26	15	+.35	6	—.22	16
8¾	—.33	17	+.32	16	—.07	11
9	—.24	13	+.24	10	—.19	8
9¼	—.30	28	+.44	37	—.23	26
9½	—.21	67	+.17	60	—.22	45
9¾	—.21	61	+.19	60	—.22	41
10	—.18	48	+.31	24	—.37	14
Stars	291	..	233	..	194	..

[1] The Milky Way is 7^h to 8^h; and 17^h to 18^h.

In Table VII the material is divided to represent the Milky Way separately. The difference between the scales is evidently about one-quarter of a magnitude outside the Milky Way. In the first branch, the difference is extremely small, up to 10 mag. In the second, the photographic estimates are one quarter brighter than the visual, for the greater part. There is thus represented a change, in the relation, of a full half magnitude.

The zone through 8^h of right ascension, in this degree, probably exhibits the photographic *DM.* at its best, and deserves some special consideration. There are but few bright stars, comparatively; the scales differ systematically less than one tenth, and the visual 10 is estimated 9.9 of the photographic scale (8^h only included); while the average difference is but ± 0.18 per star.

The actual number of stars in each list is important, and is a good criterion for each, in this case. The photographic has 208 stars, of these the visual has omitted 2 of 9.8 and 33 of magnitudes below 10.0. The visual has 205 stars, of which the photographic has omitted 1 of 9.7, 1 of 9.9, and 30 of 10. In other words, there are missing, from either list, barely more than a few stars of the faintest grades ever included. There may be made objection, that there is here no actual standard of

comparison. But this is a case for the exercise of practical judgment, rather than for the logic of figures. With two extensive lists, the portion in each representing some of the densest regions observed, each list being subject to systematic and accidental errors, and to mistakes of record and of observation, the mutual agreement indicates a degree of efficiency nearly approaching the complete purpose of the work. The photographic has to contend with the errors introduced by color, and with differences in the plates, the exposure, and in conditions of the sky. The visual has mainly to contend with the difficulties arising from the necessary rapidity of observation and estimate. The number of stars rises above 30 per minute, in this hour, for a degree in width. This distribution is, however, much exceeded in the other branch of the Milky Way.

This zone, through 8^h, is the only case in which the photographic *DM.* includes more stars in one hour than the visual, for the regions compared. From 100 per cent. in this hour, the photographic falls to 20 per cent. of the visual, in other hours of the same degree. This is a large variation ; it implies a difference of more than a whole magnitude in the scope of one list, or the other, for different portions.

Table VIII gives the number of stars, and the corresponding percentage, in each list. The percentages of the visual will be found to be in accord with Table I.

The count of stars, in the photographic evidently begins to fall off at $9\frac{1}{2}$. If we compare the count for the two at this point, it will be seen that by adding a sufficient number of stars to correspond with the systematic difference in the scales of magnitude, the photographic summation will be brought up to the visual. The photographic total is 59 per cent. of the visual, precisely the same as for the mean of the four degrees south of -34.

For the contents of Table IX, in general, no explanation is required. But the zone through 17^h and 18^h deserves treatment apart. The number of faint stars, missing from the visual, exclusive of 17^h and 18^h is three times as large as for the

TABLE VIII.

COUNT AND PERCENTAGE. -34° 20′ TO 30′.

Visual DM.					Photographic DM.				
Mag.	Stars	Sum.	Per 1000	Sum.	Mag.	Stars	Sum.	Per 1000	Sum.
U. A.	17	17	6	6	U. A.	17	17	10	10
7–8	47	64	17	23	7–8	32	49	19	29
8	21	85	8	31	8	9	58	5	34
8¼	37	122	13	44	8¼	30	88	18	52
8½	66	188	24	68	8½	72	160	44	96
8¾	77	265	28	96	8¾	75	235	46	142
9	78	343	28	124	9	59	294	36	178
9¼	149	492	54	178	9¼	145	439	89	267
9½	381	873	137	315	9½	300	739	182	449
9¾	740	1613	266	581	9¾	264	1003	161	610
10	1163	2776	419	1000	10	209	1212	127	737
					10¼	206	1418	125	862
					10½	146	1564	89	951
					10¾	81	1645	49	1000

The Uranometry stars include all to 6.9 mag.

degree preceding. There are none missing from 21^{h} to 6^{h}, inclusive.

The photographic now includes 25 per cent. of the visual 10; and it omits but one-fourth as many of the stars to visual 9¼, as in the preceding degree, and but one-third as many to 9½.

The photographic scale is sensibly different, in 17^{h} and 18^{h}, from that in any other region compared. The limit of the estimates for the full degree, from 17^{h} 29^{m} to 18^{h} 45^{m}, is 9.8, and for the last half of 17^{h} it is 9.6. At the point of division, midway of 17^{h}, the photographic scale changes one-half mag. with respect to the visual, at the same time that the limit drops from 10.2 to 9.6.

This corresponds with the omission of a comparatively large number of stars from the visual *DM.* They lie, mainly, within the limits of the cluster, 7 Messier in Scorpius, which is included in Gould's photographic clusters. But one star of those missing in this zone from the visual *DM.* is in the *Cordoba General Cata-*

logue clusters, recorded as 10 mag., and this star is not in Gould's photograph. The estimates of the photographic *DM.* are a full half magnitude brighter than the estimates of the *G. C.* cluster in this section of the zone.

TABLE IX.

STARS MISSING. —34° 20′ TO 30′.

From Vis. DM.			From Phot. DM.		
No.	Phot. mag		No.	Vis. mag.	Sum.
1	9.6	These 5 stars are in the first branch of the	1	8.5	1
4	9.8	Milky Way, 7^h to 9^h.			
			1	9.0	2
7	10.0	Of 91 stars, 10 mag. and fainter, 68 are in the	1	9.2	3
30	10¼	first branch of the Milky Way, 7^h to 9^h.	5	9.3	8
20	10½		7	9.4	15
34	10¾	Many missing stars belong to pairs.	15	9.5	30
			36	9.6	66
96		Exclusive of 17^h and 18^h.	91	9.7	157
			58	9.8	215
24	9½	In 17^h. Phot. limit 9.6.	180	9.9	395
12	9¾	In 18^h. Phot. limit 9.8.			
			868	10	1263
36		See reference to these stars in text following.			
132		Total.			

In observing these clusters with the meridian circle, estimates were made as faint as 11. I was unable to reconcile the extremely faint estimates, made by the other observers, with my own scale based upon previous practice with the larger telescope of the Albany meridian circle; but the observation of so many faint stars, at this time, probably established the uniformity of the scale down to magnitude 10.

The evidence seems to be pretty conclusive, that for this special section of the photographic *DM.* the faintest stars are recorded much too bright; and that the visual has omitted only such stars as ordinarily were below its limit.

The number of stars per degree reaches 60 for one minute of right ascension in 17^h. Such thick clusters were occasion-

ally observed by narrowing down to a zone of 20′ in the *Durchmusterung* work, making it entirely feasible to obtain all the stars that were recognized as being within its scope.

There remains the direct exhibition of the two adjoining degrees, and this is effected in the columns of Table X. The summation of the count to each quarter magnitude is given, and the difference in the scales for each type.

The two degrees of the visual are in noticeable accord with respect to the summation. The same may be said of the photographic up to 9. At 9¼ is indicated the commencement of the increase in the extension of the photographic record, which becomes more strongly marked as the faint end is approached. The scales are, however, still in accord for the same method. The systematic difference between the visual and photographic scales has virtually the same character in both degrees.

The summation of the photographic will be close to that of

TABLE X.

COMPARISON OF −33° AND −34°.

Mag.	Visual DM.		Phot. DM.		Vis.—Phot.	
	Sum. −33°	Sum. −34°	Sum. −33°	Sum. −34°	Δ Mag. −33°	Δ Mag. −34°
U. A.	17	17	17	17	——	——
7–8	64	64	47	49	−.26	−.21
8	85	85	61	58	−.19	−.17
8¼	109	122	97	88	−.14	−.23
8½	177	188	165	160	−.09	−.13
8¾	258	265	226	235	−.06	−.08
9	338	343	285	294	−.06	−.13
9¼	499	492	383	439	−.13	−.03
9½	894	873	601	739	−.14	−.11
9¾	1608	1613	770	1003	−.13	−.12
10	2757	2776	871	1212	−.18	−.19
10¼			992	1418	——	——
10½			1056	1564	−.14	−.14
10¾			1068	1645		
11			1075			

the visual, at 9 in $-33°$, and at 9½ in $-34°$, if the number of stars, proportional to the systematic difference in magnitude, be added.

A short list of errors in print, for the two volumes that have been under comparison, is given here. No mistakes have been assumed in making the identification of the stars, though occasionally the adoption of a 10′ error would have brought the places of a faint star from each list into agreement.

The two cases of 10′ discrepancies noted below may very probably be real mistakes, they are of the nature most likely to occur in the direct observation as a result of misunderstanding by the recorder, or of actual error in count by the observer. If there are two mistakes, and both in one catalogue, it is a small number to be detected in the rigorous comparison of 2500 stars.

CORRIGENDA.

CORDOBA VOL. XVII.

Page	Star	
105		Head of first column, read 15^m, in place of 14.
124	1920	read *U. A.* in place of *G. C.*
		Mag.
137	5903	9.9 The figures have
	5904	9.5 slipped down to the
	5905	7.6 following star.
	5906	9.9

PHOT. DM. VOL. I.

Omission of catalogue reference.

Page	Star	
477	3867	*Z. C.* 9
485	5830	*Z. C.* 9
514	6433	*Z. C.* 9½ but the Δ Dec. is 2′ Vis. *DM.* has two stars.
525	8763	*G. C.* 9½ but the Δ R. A. is 3^s. Precedes 7¾ closely.

10′ DISCREPANCIES.

	Visual.			Photographic.			
		Mag.	Dec.			Mag.	Dec.
157	11826	9.8	35′.4	517	6986	10.2	26′.2
167	14821	9.4	21 .3	525	8851	9.4	11 .7

As general conclusions the two scales may be stated to have a systematic difference of one quarter of a magnitude outside the Milky Way, the photographic estimates being the fainter.

This difference nearly disappears within the Milky Way and is for the following branch of $-34°$, actually reversed in sign, the photographic being one quarter brighter than the visual. The two portions of the photographic *DM.* north and south of $-34°$, have in general a consistent scale of magnitudes, but the full scope of the latter is from ¼ to ½ a magnitude greater. Completeness to the ninth magnitude is pretty well assured.

The extremely faint estimates of the scale belong, probably, to stars near the tenth magnitude, and the real extent of the photographic reproduction of the sky has much smaller variation than the difference of nearly two magnitudes (9.6 to 11.4), in the limit of estimates, would imply.

The influence of color is not greatly in evidence, unless the variation at the faint end, where it is least to be anticipated, is taken as indication.

The measurement of the plates is less likely to be influenced by systematic error in right ascension than was the direct observation. In either case the positions meet the requirements of the work with respect to accidental error and mistakes appear to be rare.

The evidence of the standing of the visual *Durchumsterung* is additional recompense for the employment of seven years in earnest and willing aid to its accomplishment.

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,
March 15, 1898.

THE CAUSTIC OF THE RIGHT PARABOLIC CYLINDER.

By C. W. CROCKETT.

LET $O\,P$ be the parabolic director, O the principal vertex, F the focus, and P any point on the arc. Let $S\,P$ be the direction of the rays of light, parallel to the plane of the parabolic director and making the angle a with the axis $O\,F$; then $P\,D$ will be the reflected ray corresponding to $S\,P$.

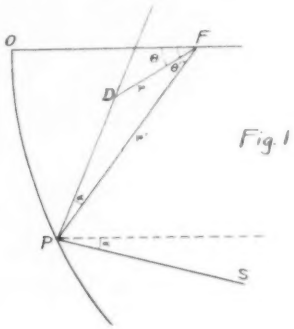

Fig. 1

With the pole at F and the initial line extending from F towards O, let

r', θ' = coördinates of P,

r, θ = coördinates of any point D on $P\,D$,

and $\frac{1}{2}\,p = O\,F$.

Then from the equation of the parabola,

$$r' = \frac{p}{1+\cos\theta'} = \tfrac{1}{2}\,p\,\sec^2\tfrac{1}{2}\,\theta' \quad - - - - - \quad (1)$$

From the triangle $P\,D\,F$,

$$r\sin(a+\theta'-\theta) = r'\sin a;$$

$$\therefore r\sin(a+\theta'-\theta) - \tfrac{1}{2}\,p\sin a\,\sec^2\tfrac{1}{2}\,\theta' = 0 \quad - \quad (2)$$

Equation (2) is the equation of the ray reflected from the point whose vectorial angle is θ'. To find the equation of the caustic, θ' must be eliminated between (2) and its first derivative with respect to θ'. This derivative is

$$r \cos (\alpha + \theta' - \theta) - \tfrac{1}{2} p \sin \alpha \sec^2 \tfrac{1}{2} \theta' \tan \tfrac{1}{2} \theta' = 0 \quad - \quad (3)$$

Substituting the value of $\tfrac{1}{2} p \sin \alpha \sec^2 \tfrac{1}{2} \theta'$ from (2) and reducing,

$$\cot (\alpha + \theta' - \theta) = \tan \tfrac{1}{2} \theta';$$

$$\therefore \quad \alpha + \theta' - \theta = 90^\circ - \tfrac{1}{2} \theta';$$

$$\therefore \quad \theta' = \tfrac{2}{3} (90^\circ + \theta - \alpha) \quad - \quad - \quad - \quad - \quad - \quad (4)$$

or

$$\theta = \tfrac{3}{2} \theta' + \alpha - 90^\circ \quad - \quad - \quad - \quad - \quad - \quad (4')$$

Substituting (4) in (2) and reducing, the equation of the caustic becomes

$$r \sin^3 (60^\circ + \tfrac{1}{3} \alpha - \tfrac{1}{3} \theta) = \tfrac{1}{2} p \sin \alpha \quad - \quad - \quad - \quad (5)$$

The coördinates of the point nearest the focus are

$$r_m = \tfrac{1}{2} p \sin \alpha, \; \theta_m = \alpha - 90^\circ \quad - \quad - \quad - \quad - \quad - \quad - \quad (6)$$

If the vectorial angles are measured from this minimum radius vector instead of from the axis, and if θ_1 represents the new vectorial angle,

$$\theta_1 = 90^\circ + \theta - \alpha \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad (7)$$

and (5) becomes

$$r = \tfrac{1}{2} \frac{p \sin \alpha}{\cos^3 \tfrac{1}{3} \theta_1} \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad (8)$$

showing that the caustic is symmetrical with respect to the minimum radius vector.

From (4′) and (7)

$$\theta_1 = \tfrac{3}{2} \theta' \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad (9)$$

The length of the arc of the caustic corresponding to a mirror with the angular aperture $2\theta'$ is

$$s = 3 p \sin \alpha (\tan \tfrac{1}{2} \theta' + \tfrac{1}{3} \tan^3 \tfrac{1}{2} \theta') \quad - \quad - \quad (10)$$

Fig. 2 shows the portion of the caustic that falls within the mirror when $\alpha = 30^\circ$ and the aperture is 240°, the degrees marked along the caustic being the vectorial angles of the corresponding points on the parabola.

In Fig. 3, the angular aperture $2\theta'$ is 40°, and $a\,o\,b$ is the caustic formed when $\alpha = 30^\circ$, a corresponding to A, o to O, and b to B. If α is less than 30° the points a, o and b will fall

nearer the focus, the arc $a\,o\,b$ will be shorter, and the points a, o and b will describe the circular arcs $a\,F$, $o\,F$, and $b\,F$ respectively as α approaches zero.

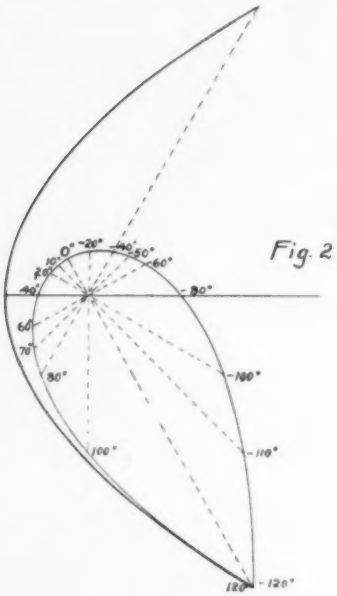

Fig. 2

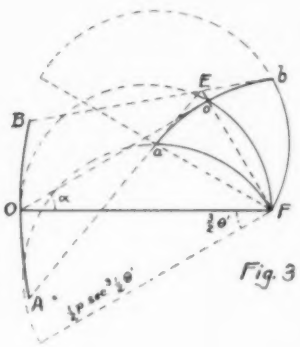

Fig. 3

The extreme rays $A\,a$ and $B\,b$ (Fig. 3) meet the line $F\,o$ at E and the cross section of the system of rays is $o\,E$. The coördinates of the point o are $(\frac{1}{2}\,p \sin \alpha,\ \alpha - 90°)$. The direction angle of E is also $\alpha - 90°$; substituting this for θ in (2) gives

$$F\,E = \tfrac{1}{2}\,\frac{p \sin \alpha}{\cos^2 \tfrac{1}{2}\,\theta' \cos \theta'}.$$

$$\therefore\ o\,E = F\,E - F\,o = \tfrac{1}{2}\,p \sin \alpha \left(\frac{1}{\cos^2 \tfrac{1}{2}\,\theta' \cos \theta'} - 1\right)$$

which reduces to

$$o\,E = \tfrac{1}{2}\,p \sin \alpha \tan^2 \tfrac{1}{2}\,\theta' \,\frac{2 + \cos \theta'}{\cos \theta'}.$$

If the angle at the vertex of the mirror subtended by $o\,E$ (*i. e.*, $o\,O\,E$) is represented by γ,

$$\tan \gamma = \frac{o E}{O o} = \tan \alpha \tan^2 \tfrac{1}{2} \theta' \frac{2 + \cos \theta'}{\cos \theta'} \quad - \quad - \quad (11)$$

equivalent to the final expression for tan γ derived by Dr. Poor.[1] This applies, however, to the parabolic cylinder and not to the paraboloid of revolution.

RENSSELAER POLYTECHNIC INSTITUTE,
March 24, 1898.

[1] This JOURNAL, February 1898, p. 119 and *Ast. Jour.*, **420**, p. 90.

THE PARABOLIC MIRROR.

By C. W. Crockett.

Let O-APB be a paraboloid of revolution, O the principal vertex, F the focus, P any point on the circle APB whose plane is perpendicular to the axis, and PN the normal at P.

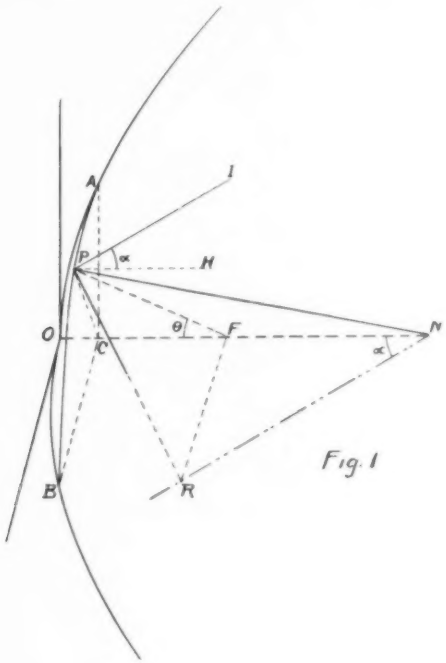

Fig. 1

Let the incident rays, such as IP, be parallel to the vertical plane AOF and make the angle α with the horizontal plane BOF, and let PR be the reflected ray corresponding to IP. Then the ray reflected from any point P on the circle APB must pierce the vertical plane AOF somewhere in the line NR drawn through N parallel to IP; for if a plane be passed through IP parallel to the vertical plane AOF, the plane IPR,

containing IP, PN and PR, must intersect the parallel planes in the parallel lines IP and NR.

Let $v = \sphericalangle IPN = \sphericalangle NPR$,
$r = FP$,
$\theta = OFP$,
$p = CN$,
$\frac{1}{2}p = OF$,
ψ = angle between the vertical plane AOF and the plane PCN through the axis ON and P,
$= \sphericalangle ACP$.

I. In the plane triangle PFN, $PF = FN$.

$\therefore \sphericalangle FPN = \sphericalangle FNP = \frac{1}{2}\theta$.

$\therefore PN = p \sec \frac{1}{2}\theta$.

II. Constructing a sphere about P as a center, the planes $IPNR$ and $HPNF$ cut two intersecting arcs of great circles, which with the arcs cut by the planes IPH and FPR form two congruent spherical triangles, from which we find that the plane angle $FPR = \alpha$ and the dihedral angle $NPF\text{-}RPF = 180° - \psi$.

III. Constructing a sphere about N, the planes FNP, FNR, and PNR give a spherical triangle having two sides and the included angle equal to two sides and the included angle of the triangles formed when the sphere was passed about P.

$\therefore$ Plane angle $PNR = v$.

Hence in the plane triangle PNR, we have

$\sphericalangle PNR = \sphericalangle NPR$.

$\therefore RP = RN$.

IV. In the plane triangles FRP and FRN, it has been shown that

$$FP = FN;\ RP = RN;\ \sphericalangle RPF = \sphericalangle RNF = \alpha.$$

Hence the two triangles are congruent, and the triangle FPR may be brought into coincidence with FNR by revolution about FR.

V .Pass a plane DFN (Fig. 2) through the axis, making an angle ψ_1 with the vertical plane AOF, and represent the plane angle NFD by θ_1 and the radius vector FD by r_1. Then from the plane triangle PFD,

$$r \sin a = r_1 \sin FDP = r_1 \sin (DFP + a).$$

$$\therefore \quad r_1 = r \sin a \operatorname{cosec} (\gamma + a) \quad - - - - - - \quad (1)$$

where $\gamma = DFP$.

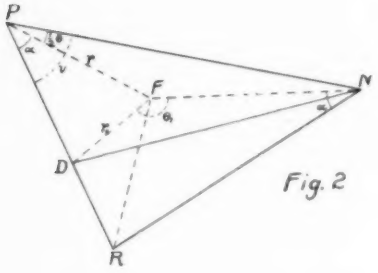

Fig. 2

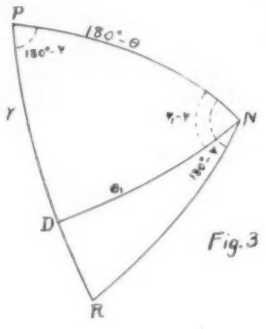

Fig. 3

VI. Constructing a sphere about F, the planes PFN, DFN, RFN and PFR will cut the surface in a figure, similar to that shown in Fig. 3, where

$$DP = \gamma, \; PN = 180° - \theta, \; DN = \theta_1, \; DNP = \psi_1 - \psi,$$
$$RNP = 180° - \psi = RPN$$

(shown in Section II). Hence

$$\sin \gamma = \frac{\sin (\psi_1 - \psi)}{\sin \psi} \sin \theta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\cos \gamma = - \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos (\psi_1 - \psi) \quad . \quad (3)$$

$$\sin \theta \cot \theta_1 = - \cot \psi \sin (\psi_1 - \psi) - \cos \theta \cos (\psi_1 - \psi) \quad (4)$$

VII. To find the point where the reflected ray pierces the plane through the focus perpendicular to the axis, make $\theta_1 = 90°$ in Eqs. (1), (2), (3) and (4).

$$\left.\begin{aligned} \therefore \tan(\psi_1-\psi) &= -\cos\theta\tan\psi \\ \sin\gamma &= \frac{\sin(\psi_1-\psi)}{\sin\psi} \\ \cos\gamma &= \sin\theta\cos(\psi_1-\psi) \\ r_1 &= \frac{r\sin a}{\sin(\gamma+a)} \end{aligned}\right\} \quad \ldots \quad (5)$$

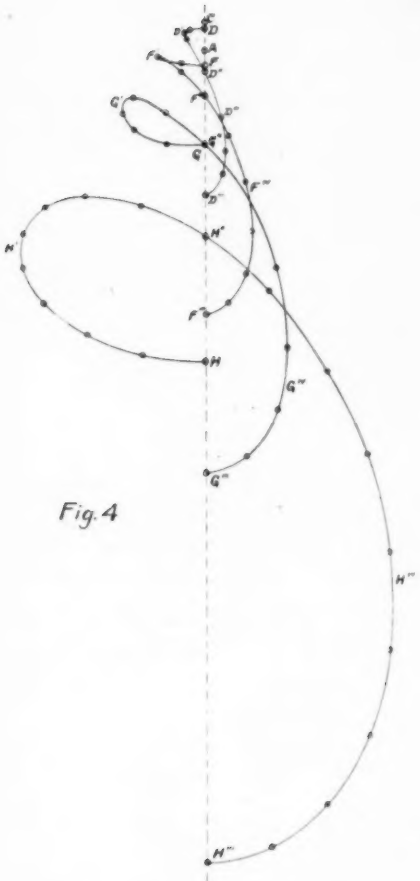

Fig. 4

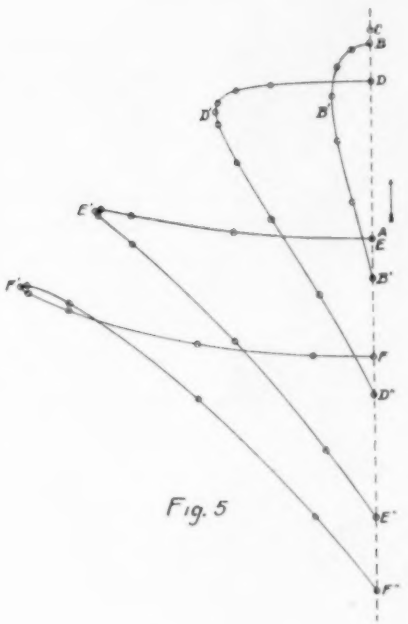

Fig. 5

VIII. When θ is less than 90°—a, the following conditions must be satisfied in Eqs. (5):

for $\psi < 90°$; $\psi_1 > 180°$, $\psi_1 - \psi$ between 90° and 270°;
for $\psi > 90°$; $\psi_1 < 180°$, $\psi_1 - \psi$ numerically less than 90°.

Conclusion.—Assuming that $a = 1°$ and considering the circles on the paraboloid such that θ has the values 0°, 0°30′, 0°40′, 1°00′, 1°20′, 1°30′, 2° and 3°, Eqs. (5) lead to the results shown in Figs. 4 and 5, the latter figure showing the upper part of Fig. 4 on a larger scale. The plane of the paper is the focal plane, the vertical line FA passes through the focus which is above A at the distance $\frac{1}{2}p \tan 1°$, and the vertex of the paraboloid is behind the paper, so that the observer is looking towards the vertex. In Fig. 4 the distance $AH = 0.0000199 \times \frac{1}{2}p$ and in Fig. 5 $AF'' = 0.0000029 \times \frac{1}{2}p$. The diagrams show the results corresponding to ψ from 0° to 180°—*i. e.*, for half the mirror—and the curves should therefore be symmetrical with reference to the line FA.

The traces are as follows:

for $\theta = 0°$, the trace is the point A;
$\theta = 0°30'$, the trace is the curve $B''' B'' B' B$ (Fig. 5);
$\theta = 0°40'$, the highest point of the image is at C;
$\theta = 1°00'$, the trace is the curve $D''' D'' D' D$;
$\theta = 1°20'$, the trace is the curve $E'' E' E$ (Fig. 5);
$\theta = 1°30'$, the trace is the curve $F''' F'' F' F$;
$\theta = 2°00'$, the trace is the curve $G''' G'' G' G$;
$\theta = 3°00'$, the trace is the curve $H''' H'' H' H$.

The curves were constructed from the points marked in the diagrams, as they indicate the forms of the curves with an accuracy sufficient for the purpose of this article.

It will be noticed that as θ increases from 0°, the trace starting as the point A becomes a curve similar to B, the highest point moving upwards until it reaches C; then the trace becomes similar to D, the highest point that lies on the line FA moving downward, in the E curve coinciding with A. The loop first appears when $\theta = 1°25'$ approximately, increasing in size through the form of the F curve to that of the G, after which the trace is similar to H.

RENSSELAER POLYTECHNIC INSTITUTE,
March 24, 1898.

COMPARISON OF OXYGEN WITH THE EXTRA LINES IN THE SPECTRA OF THE HELIUM STARS, β CRUCIS, ETC., ALSO SUMMARY OF THE SPECTRA OF SOUTHERN STARS TO THE 3½ MAGNITUDE AND THEIR DISTRIBUTION.[1]

By FRANK MCCLEAN.

IN a previous paper read before the Society on April 8, 1897, I suggested that the special lines present in spectra of the first division of helium stars (Type I, Division I*a*) might possibly be due to oxygen. These stars are associated by their position and distribution with the gaseous nebulæ, and some of the lines in their spectra correspond with bright lines observed by Campbell in nebulæ. The suggestion from this was that these stars are in the first stage of stellar development from gaseous nebulæ.

The special lines referred to are the extra lines which distinguished these spectra from those of the remaining helium stars of Division I*b*.

The indications in the spectra of the northern stars that these extra lines are due to oxygen are slight, as the lines at best are indistinct. Among the southern stars, however, there are several in the spectra of which these lines are better defined, and there is one, viz., β Crucis, in which they are very fairly defined.

The following stellar spectra are mounted on the plate presented to the Society, viz., κ Orionis, β Scorpii, β Canis Majoris, β Centauri, and β Crucis. These photographs are intended to show the gradual improvement in the definition of the extra lines, between κ Orionis and β Crucis, and to indicate their identity of origin throughout.

The extra lines in the spectrum of β Crucis are singled out by comparison with another helium star, viz., κ Argus, of Division I*b*, in which the extra lines do not appear. The lines are drawn out by themselves below the spectrum of β Crucis. They are then compared directly by juxtaposition with a drawing of the spectrum of oxygen as tabulated in the spectrum of air by Neovius (Stockholm, 1891, and Appendix E, 1894, of *Watts's Index*).

This comparison shows a close correspondence in the grouping of

[1] Read before the Royal Society.

the extra lines with the spectrum of oxygen. The most remarkable correspondence is in the case of the large group on either side of $H\delta$. A slight shift of about a tenth-meter is required to bring the groups into identical positions. However, the close similarity of the whole grouping of the two spectra as they appear on the plate admits of little doubt that the extra lines actually constitute the spectrum of oxygen. If this be established the spectrum of the first division of helium stars would be due to hydrogen, helium and oxygen.

The scale attached to the spectra is based on standard lines that can be identified with certainty in the stellar spectra. It is interpolated between the standard lines. Its position in relation to the spectra is determined by the hydrogen lines. The wave-lengths employed are in accordance with Ångström's scale.

On the original negatives the distance between (H) and (F) measures about 1 inch. The negatives are enlarged about eight and a half times. It is difficult to fix the position of the lines — and especially of the hydrogen lines — on these enlargements with sufficient accuracy. A further correction than this would account for is, however, required in order to reduce the two spectra to exact coincidence. I believe it should be sought to some extent in a reëxamination of the adopted wave-lengths of the hydrogen and of the oxygen spectra.

The spectrum of γ Argus is given on the plate in order to identify it as a helium star. It contains two crucial lines of helium. The Wolf-Rayet stars, of which it is the principal example, are thus classified as helium stars. There are also some coincidences between the bright lines of γ Argus and the spectrum of oxygen, which suggest a possible connection.

The spectrum of μ Centauri is also given as a bright line helium star. The bright lines in this case are due to hydrogen, and the spectrum resembles that of γ Cassiopeiæ. The spectrum of γ Centauri is similar.

I take this opportunity of presenting a summary of the spectra of 116 stars to the $3\frac{1}{2}$ magnitude in the Southern Hemisphere. They were photographed between May and October last by means of my own object-glass prism, mounted in front of the Cape astrographic telescope. This instrument, which is similar to my own telescope at Rusthall, with which the spectra of the northern stars were photographed, was kindly placed at my disposal by H. M. Astronomer, Dr. Gill. It may be a little time before the actual photographs of the stellar

spectra are ready for presentation, and meanwhile the results are of interest.

In my previous paper I divided the sphere into eight equal areas consisting of two galactic equatorial areas and two galactic polar areas, situated on either side of the galactic equator. The northern stars

TABLE I.

PHOTOGRAPHIC STELLAR SPECTRA — STARS TO MAGNITUDE 3½. SUMMARY OF SOUTHERN STARS — REGIONS BB, CC, AND DD.

	Mag.	Div.	Area		Mag.	Div.	Area
Aquila				Centaurus			
λ	3.3	I (*b*)	CC	α	0.7	IV	CC
Ara				β	1.2	I (*a*)	BB
α	2.9	I (*b*)	CC	γ	2.4	I (*b*)	BB
β	2.8	IV	CC	δ	2.8	II	BB
γ	3.6	I (*a*)	CC	ε	2.6	I (*a*)	BB
ζ	3.2	IV	CC	ζ	2.7	I (*b*)	BB
Argo				η	2.5	I (*b*)	BB
α	0.4	III	CC	θ	2.7	IV	BB
β	2.0	II	CC	ι	3.0	III	BB
γ	3.0	I (*b*)	CC	κ	3.3	I (*b*)	BB
δ	2.2	II	CC	λ	3.4	I (*b*)	CC
ε	2.1	IV	CC	μ	3.4	I (*a*)	BB
ζ	2.5	I (*b*)	CC	Circinus			
θ	2.9	I (*a*)	CC	α	3.5	III	CC
ι	2.5	III	CC	Columba			
κ	2.7	I (*b*)	CC	α	2.7	I (*b*)	CC
λ	2.5	IV	BB	β	2.9	IV	CC
μ	2.9	IV	BB				
ν	3.5	I (*b*)	CC	Crux			
ξ	3.4	IV	BB	α	1.3	I (*a*)	BB
π	2.7	IV	CC	β	1.7	I (*a*)	BB
ρ	3.2	III	BB	γ	2.0	V	BB
σ	3.5	IV	CC	δ	3.4	I (*b*)	BB
τ	3.2	IV	CC				
υ	3.4	III	CC	Doradus			
				α	3.1	I (*b*)	DD
Canis Major				Eridanus			
α	−1.4	II	CC	α	1.0	I (*b*)	DD
β	2.0	I (*a*)	CC	θ	2.6	II	DD
δ	1.9	IV	CC	φ	3.5	I (*b*)	DD
ε	1.5	I (*a*)	CC	χ	3.9	IV	DD
ζ	3.0	I (*b*)	CC	Grus			
η	2.4	I (*b*)	CC	α	1.9	I (*b*)	DD
o^2	3.0	I (*b*)	CC	β	2.2	V	DD
				γ	3.0	I (*b*)	DD
Carpricornus				ε	3.5	II	DD
β	3.4	IV	CC				

TABLE I — *Continued.*

	Mag.	Div.	Area		Mag.	Div.	Area
Hydrus				Piscis Austr.			
α	2.9	III	DD	α	1.3	II	DD
β	2.7	IV	DD				
γ	3.2	V	DD	Recticulum			
				α	3.3	IV	DD
Indus							
α	3.1	IV	DD	Sagittarius			
				γ^2	3.0	IV	CC
Lepus				δ	2.8	IV	CC
α	2.7	III	CC	ε	2.1	I (*b*)	CC
β	3.0	IV	CC	ζ	2.9	II	CC
ε	3.3	IV	CC	η	3.0	V	CC
μ	3.3	I (*b*)	CC	λ	3.1	IV	CC
				π	3.1	III	CC
Libra				σ	2.3	I (*b*)	CC
σ (20)	3.2	V	BB	φ	3.3	I (*b*)	CC
Lupus				Scorpius			
α	2.6	I (*a*)	BB	α	1.1	V	BB
β	2.8	I (*a*)	BB	β′	2.9	I (*a*)	BB
γ	3.2	I (*a*)	BB	δ	2.5	I (*a*)	BB
δ	3.7	I (*a*)	BB	ε	2.2	IV	BB
ε	3.7	I (*b*)	BB	θ	2.1	III	CC
				ι	3.3	III	CC
Musca				κ	2.6	I (*a*)	CC
α	2.9	I (*b*)	CC	λ	1.7	I (*a*)	CC
β	3.4	I (*b*)	CC	μ	3.6	I (*a*)	BB
				π	3.1	I (*a*)	BB
Ophiuchus				σ	3.0	I (*a*)	BB
β	2.9	IV	BB	τ	2.9	I (*a*)	BB
ζ	2.8	I (*a*)	BB	υ	2.8	I (*b*)	CC
η	2.6	II	BB				
θ	3.4	I (*b*)	BB	Serpens			
κ	3.4	IV	BB	η	3.4	IV	BB
Pavo				Telescopium			
α	2.1	I (*b*)	DD	α	3.5	I (*b*)	CC
β	3.3	III	DD				
δ	3.5	IV	DD	Toucan			
				α	2.8	IV	DD
Phœnix				Triangulum			
α	2.4	IV	DD	α	2.2	IV	CC
β	3.3	IV	DD	β	3.1	III	CC
γ	3.4	IV	DD	γ	3.1	II	CC

NOTE.—The magnitudes are taken from the *Nautical Almanac* (or from Gould).

already given occupy the upper or northerly lateral areas A, B, C and D, also the southerly area AA. The southern stars now given occupy the lower or southerly lateral areas BB, CC and DD. Their photo-

graphic spectra are distributed into these areas, and are classified on the same system as in the previous paper. The table of distribution for the whole sphere by areas and classes is given below.

TABLE II.

SUMMARY TABLES OF DISTRIBUTION OF GASEOUS NEBULÆ AND OF STELLAR TYPES. STARS TO THE 3½ MAGNITUDE.

No. 1.

Stellar types	A	B	C	D	Total	AA	BB	CC	DD	Total
Planetary nebulæ	2	3	8	2	(15)	2	7	3	0	(12)
Extended nebulæ.....	1	4	8	4	(17)	1	4	3	1	(9)
Total gaseous nebulæ.	3	7	16	6	(32)	3	11	6	1	(21)

NOTE.—Extracted from table in Frost's edition of *Scheiner's Astronomical Spectroscopy*.

No. 2.

Stellar types	A	B	C	D	Total	AA	BB	CC	DD	Total
Division I.....	3	6	17	3	(29)	6	23	25	6	(60)
" II.....	10	7	0	3	(20)	3	2	5	3	(13)
" III.....	7	8	8	4	(27)	9	1	9	2	(21)
" IV.....	14	8	9	13	(44)	9	9	16	9	(43)
" V.....	1	2	4	3	(10)	3	3	1	2	(9)
	35	31	38	26	(130)	30	38	56	22	(146)

No. 3.

Stellar types	A	B	C	D	Total	AA	BB	CC	DD	Total
Division I.....	3	6	17	3	(29)	6	23	25	6	(60)
" II and III.....	17	15	8	7	(47)	12	3	14	5	(34)
" IV and V.....	15	10	13	16	(54)	12	12	17	11	(52)
	35	31	38	26	(130)	30	38	56	22	(146)

No. 4.

Stellar types	A and AA	B and BB	C and CC	D and DD	Total
Division I.....	9	29	42	9	(89)
" II and III.....	29	18	22	12	(81)
" IV and V.....	27	22	30	27	(106)
	65	69	94	48	(276)

There are in all 89 helium stars (Division I), distributed 71 in the galactic zones and 18 in the galactic polar areas, the areas being equal. There are 29 in the upper galactic zone (B and BB), and 42 in the lower galactic zone (C and CC). There are 9 in the upper polar areas (A and AA), and 9 in the lower polar areas (D and DD). There are 23 in the northerly halves of the two galactic zones (B and C) and 48 in the southerly halves (BB and CC).

The 81 stars in Division II, the Sirian stars, and Division III, the Procyon stars (which along with Division I constitute Secchi's Type I) are rather irregularly distributed throughout the sphere. There are 40 in the galactic zones and 41 in the galactic polar areas. There are 18 in the upper galactic zone (B and BB) and 22 in the lower (C and CC). There are 29 in the upper polar areas (A and AA) and 12 in the lower (D and DD). To the extent of the observations there is no condensation of stars of Divisions II and III in the galactic zones as there is in the case of stars of Division I.

The 106 stars in Divisions IV and V (II and III of Secchi's types) are fairly evenly distributed throughout the sphere. There are 52 in the galactic zones and 54 in the galactic polar areas. There are 22 in the upper galactic zone (B and BB) and 30 in the lower (C and CC). There are 27 in the upper polar areas (A and AA) and 27 in the lower (D and DD).

The general distribution of the types of spectra throughout the sphere to the extent of the observations bears out generally the conclusion that stars with spectra of the more advanced types, in order of development, are evenly distributed in space. Also that stars with spectra more recent in order of development are mostly congregated in the galactic zones. The helium stars of Division I are predominant in the Southern Hemisphere, being congregated in the lower or southerly halves of the galactic zones (BB and CC). They include 48 stars out of a total of 94 stars in those areas. They are also more closely congregated in the vicinity of the galaxy than is the case in the northerly halves of the galactic zones. In the contiguous constellations of Musca, Crux, Centaurus, Lupus and Scorpius, there are 27 helium stars out of a total of 36 stars included in the tables. (The distribution of the helium stars throughout the sphere was illustrated by two small hand charts, not reproduced, on which these stars are colored red.) Apparently the region in which the first stage of stellar development is now most active lies in the southerly half of the galaxy.

ARC-SPECTRA OF ZIRCONIUM AND LANTHANUM.[1]

By HENRY A. ROWLAND and CALEB N. HARRISON.

SOLAR LINES FOR STANDARDS.

PLATE 32.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3050.217	.212*	+.005	3218.399	.390‖	+.009
3053.180	.173‖	+.007	3224.382	.368‖	+.014
3055.823	.821*	+.002	3232.412	.404‖	+.008
3061.931	.930‖	+.001	3260.393	.384‖	+.009
3077.303	.303‡	.000	3267.840	.839‖	+.001
3078.145	.148‖	—.003	3274.096	.092†	+.004
3086.900	.891‖	+.009	3287.795	.791†	+.004
3094.740	.739‖	+.001	3292.174	.174†	.000
3095.013	.003‖	+.010	3295.954	.957‖	—.003
3115.166	.160‖	+.006	3302.505	.501†	+.004
3121.284	.275†	+.009	3306.475	.471†	+.004
3129.900	.882‖	+.008	3308.925	.928†	—.003
3137.456	.441‖	+.015	3318.158	.163†	—.005
3140.887	.869‖	+.018	3331.750	.741†	+.009
3153.882	.870‖	+.012	3351.871	.877‖	—.006
3167.299	.290‖	+.009	3356.215	.222‖	—.007
3172.187	.175†	+.012	3405.230	.272†	—.042
3176.118	.104‖	+.014	3406.549	.581‖	—.032
3188.173	.164‖	+.009	3406.916	.955‖	—.039
3200.033	.032‖	+.001			

PLATE 36.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3406.550	.581‖	—.031	3486.029	.036†	—.007
3406.920	.955‖	—.035	3490.704	.721*	—.017
3425.706	.721‖	—.015	3491.464	.464‖	.000
3440.730	.759*	—.029	3500.993	.993†	.000
3441.125	.135*	—.010	3510.990	.987‖	+ 003
3456.368	.384‖	—.016	3518.487	.487†	.000
3465.972	.991†	—.019	3521.404	.404†	.000
3475.578	.594†	—.016	3540.263	.266†	—.003

[1] Attention is called to the fact that in the following tables the symbols *, †, ‡, ‖, represent the relative weights assigned to standards, and are in ascending scale of magnitude.

PLATE 36.—ZIRCONIUM—*continued.*

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3545.339	.333‡	+.006	3652.677	.692†	—.015
3549.149	.145	+.004	3658.686	.688†	—.002
3550.003	.000	—.003	3667.387	.397	—.010
3558.662	.670*	—.008	3680.047	.064*	—.017
3564.680	.680†	.000	3683.186	.202†	—.016
3570.228	.225*	+.003	3684.247	.259†	—.012
3581.335	.344*	—.009	3695.178	.194†	—.016
3583.478	.483‡	—.005	3707.170	.186†	—.016
3597.188	.192	—.004	3716.578	.585†	—.007
3609.009	.015*	—.006	3727.766	.763†	+.003
3612.214	.217	—.003	3732.528	.542†	—.014
3618.920	.924*	—.004	3747.075	.095*	—.020
3623.325	.332	—.007	3780.829	.846‡	—.017
3623.598	.603	—.005	3781.309	.330	—.021
3631.615	.619*	—.004	3783.656	.674†	—.018
3640.530	.536†	—.006	3788.058	.032*	+.026
3647.997	.995*	+.002			

PLATE 40.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3883.757	.773‡	—.016	4016.578	.578†	.000
3897.589	.599	—.010	4029.792	.796	—.004
3905.657	.666*	—.009	4034.638	.641†	—.003
3924.663	.669‡	—.006	4045.974	.975*	—.001
3925.335	.345‡	—.010	4048.888	.893†	—.005
3926.123	.123†	.000	4055.700	.701‡	—.001
3937.470	.474†	—.004	4062.601	.602‡	—.001
3941.020	.021‡	—.001	4063.750	.756*	—.006
3942.556	.559†	—.003	4073.918	.920	—.002
3950.097	.101‡	—.004	4083.755	.767†	—.012
3950.485	.497	—.012	4083.925	.928	—.003
3954.001	.001	.000	4103.095	.101†	—.006
3957.180	.180*	.000	4107.642	.646†	—.004
3960.430	.429	+.001	4114.602	.600	+.002
3971.478	.478	.000	4121.476	.481	—.005
3977.895	.891	+.004	4121.907		
3981.916	.914‡	+.002	4157.936	.948	—.012
3984.080	.078†	+.002	4185.045	.063	—.018
3986.906	.903*	+.003	4197.238	.251‡	—.013
4016.572	.578	—.006			

PLATE 44.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4283.148	.170†	—.022	4415.289	.299†	—.010
4289.504	.523†	—.019	4425.607	.609†	—.002
4293.216	.249*	—.033	4447.889	.899	—.010
4222.339	.381	—.042	4454.936	.950†	—.014
4226.848	.892†	—.044	4494.725	.735†	—.010†
4250.924	.956†	—.032	4497.026	.041†	—.015
4302.680	.689†	—.009	4499.063	.070†	—.007
4308.064	.034*	+.030	4501.438	.444‡	—.006
4343.382	.387†	—.005	4508.449	.456	—.007
4352.895	.903	—.008	4554.187	.213†	—.026
4359.770	.778†	—.008	4571.265	.277†	—.012
4369.930	.943	—.013	4572.142	.157†	—.015
4391.140	.149	—.009	4578.720	.731‡	—.011
4404.917	.927†	—.010	4588.369	.384	—.015
4407.838	.850†	—.012	4590.112	.129	—.017

PLATE 48.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4611.455	.453*	+.002*	4805.263	.253†	+.010
4629.509	.515†	—.006†	4810.732	.723†	+.009
4637.683	.683	.000	4823.705	.697†	+.008
4638.189	.194‡	—.005	4824.334	.325	+.009
4668.300	.303*	—.003	4859.939	.934‡	+.005
4678.345	.353†	—.008	4883.876	.867†	+.009
4686.392	.395†	—.003	4893.045	.030†	+.015
4690.325	.324†	+.001	4917.410	.410†	.000
4691.574	.581*	—.007	4924.104	.109	—.005
4703.181	.180*	+.001*	4934 248	.247*	+.001
4722.355	.349‡	+.006	4936.005	.015†	—.010
4727.628	.628†	.000	4991.221	.247†	—.026
4754.230	.226†	+.004	4994.292	.316	—.024

THE ARC-SPECTRUM OF ZIRCONIUM.

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
3054.929	—.002	10	3054.927	3205.028	—.012	3	3205.016
3060.220	—.002	5	3060.218	3208.444	—.012	3	3208.432
3061.451	—.002	3	3061.449	3212.135	—.012	5	3212.123
3064.748	—.002	4	3064.746	3212.974	—.012	3	3212.962
3065.315	—.002	3	3065.313	3214.318	—.012	7	3214.306
3072.240	—.002	1 n	3072.238	3228.933	—.011	7	3228.922
3085.465	—.004	2	3085.461	3234.250	—.010	7	3234.240
3094.914	—.006	2	3094.908	3235.884	—.010	1	3235.874
3095.180	—.006	8	3095.174	3241.180	—.009	7	3241.171
3095.441	—.006	1	3095.435	3244.116	—.008	1	3244.108
3099.331	—.007	10	3099.324	3247.680	—.008	1	3247.672
3109.453	—.009	1 n	3109.444	3250.570	—.007	7	3250.563
3106.682	—.008	10	3106.674	3260.245	—.005	3	3260.240
3110.654	—.009	1	3110.645	3264.949	—.004	3	3264.945
3110.980	—.009	10	3110.971	3269.791	—.003	7	3269.788
3119.330	—.010	1	3119.320	3272.335	—.002	7	3272.333
3120.861	—.010	8	3120.851	3273.170	—.002	7	3273.168
3125.319	—.011	4	3125.308	3279.400	—.001	10	3279.399
3126.021	—.011	10	3126.010	3280.601	—.001	1 n	3280.600
3129.286	—.011	10	3129.275	3282.969	—.000	2	3282.969
3129.865	—.011	10	3129.854	3284.827	—.000	10	3284.827
3132.177	—.011	7	3132.166	3286.025	—.000	2	3286.025
3133.335	—.011	2	3133.324	3288.934	+.001	5	3288.935
3133.595	—.011	8	3133.584	3306.409	+.002	7	3306.411
3137.083	—.011	4	3137.072	3310.022	+.002	3	3310.024
3138.775	—.011	10	3138.764	3311.480	+.002	2	3311.482
3149.937	—.012	3	3149.925	3313.830	+.002	5	3313.832
3155.792	—.012	7	3155.780	3314.613	+.002	7	3314.615
3157.108	—.012	7	3157.096	3316.323	+.002	1	3316.325
3157.944	—.012	7	3157.932	3318.640	+.002	4	3318.642
3164.423	—.012	10	3164.411	3319.146	+.002	7	3319.148
3165.570	—.012	7	3165.558	3323.115	—.001	7	3323.114
3166.076	—.012	7	3166.064	3326.545	—.001	1	3326.544
3166.387	—.012	6	3166.375	3334.381	+.001	7	3334.382
3166.749	—.012	1	3166.837	3334.743	+.001	7	3334.744
3178.205	—.012	7	3178.193	3338.543	+.002	7	3338.545
3185.183	—.012	8	3185.171	3340.611	+.003	7	3340.614
3182.050	—.012	7	3182.038	3344.913	+.004	7	3344.917
3182.965	—.012	10	3182.953	3353.775	+.008	2	3353.783
3191.340	—.012	7	3191.328	3376.375	+.020	5	3376.395
3192.024	—.012	7	3192.012	3377.565	+.020	5	3377.585
3194.548	—.012	1	3194.536	3380.038	+.022	1	3380.060
3196.844	—.012	1	3196.832	3387.976	+.026	7	3388.002
3204.485	—.012	3	3204.473	3388.345	+.026	7	3388.371

ZIRCONIUM — *continued*.

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
3392.350	+.028	10	3392.378	3663.778	+.006	8	3663.784
3393.239	+.029	7	3393.268	3671.403	+.007	2	3671.410
3394.360	+.029	1	3394.389	3674.852	+.007	3	3674.859
3396.438	+.030	7	3396.468	3680.666	+.008	5	3680.674
3398.965	+.032	7	3398.997	3691.521	+.009	8	3691.530
3402.977	+.034	5	3403.011	3696.421	+.009	1 ?	3696.430
3403.806	+.034	1	3403.840	3697.593	+.010	2	3697.603
3404.955	+.033	5	3404.987†	3891.504	+.012	5	3891.516
3407.595	+.032	1 n	3407.627	3892.149	+.012	1, n	3892.161
3408.198	+.037	7	3408.235	3896.653	+.011	2	3896.664
3410.374	+.031	1	3410.405	3897.788	+.010	1	3897.798
3410.328	+.038	7	3410.366	3900.639	+.010	5	3900.649
3414.787	+.029	1	3414.816	3916.060	+.007	5	3916.067
3419.786	+.027	1	3419.813	3921.923	+.005	5	3921.928
3424.953	+.025	1	3424.978	3929.662	+.004	6	3929.666
3430.661	+.022	3	3430.683	3934.250	+.004	4	3934.254
3437.271	+.020	2	3437.291	3934.915	+.004	4	3934.919
3438.361	+.020	5	3438.381	3936.188	+.003	2	3936.191
3446.754	+.016	3	3446.770	3941.756	+.002	2	3941.758
3447.498	+.016	4	3447.514	3958.353	+.001	8	3958.354
3456.042	+.014	3	3456.056	3966.417	.000	1, n	3966.417
3457.327	+.013	2	3457.340	3973.549	.000	1	3973.549
3457.702	+.013	3	3457.715	3975.435	—.001	4	3975.434
3459.071	+.012	1	3459.083	3977.422	—.001	3	3977.421
3461.232	+.012	3	3461.244	3979.375	—.001	3	3979.374
3463.155	+.011	3	3463.166	3981.727	—.001	10	3981.726
3471.321	+.008	6	3471.329	3982.305	—.001	2	3982.304
3478.448	+.007	1	3478.455	3991.269	—.001	8	3991.268
3478.923	+.006	3	3478.929	4012.395	.000	4	4012.395
3481.295	+.005	5	3481.300	4018.520	.000	3	4018.520
3482.944	+.005	3	3482.949	4024.586	.000	3	4024.586
3483 674	+.005	3	3483.679	4025.060	.000	5	4025.060
3496.340	+.003	7	3496.343	4027.349	+.001	5	4027.350
3499.725	+.002	1	3499.727	4028.098	+.001	3	4028.099
3505.581	+.001	1	3505.582	4029.820	+.001	5	4029.821
3505.812	+.001	4	3505.813	4030.187	+.001	4	4030.188
3519.735	+.001	10	3519.736	4031.496	+.001	1	4031.497
3552.092	+.001	3	3552.093	4032.196	+.001	4	4032.197
3556.743	+.001	5	3556.744	4032.210	+.001	1	4032.211
3558.942	+.002	1	3558.944	4034.230	+.001	3	4034.231
3572.603	+.003	10	3572.606	4036.038	+.001	4	4036.039
3601.326	+.005	10	3601.331	4040.386	+.002	3	4040.388
3612.037	+.005	1	3612.042	4041.787	+.002	3	4041.789
3613.242	+.005	3	3613.247	4042.371	+.002	3	4042.373
3614.920	+.005	5	3614.925	4043.720	+.002	5	4043.722
3624.000	+.005	8	3624.005	4045.756	+.002	6	4045.758
3630.164	+.005	1	3630.169	4045.970	+.002	3	4045.972
3634.293	+.005	3	3634.298	4048.811	+.002	7	4048.813
3636.596	+.005	2	3636.601	4049.714	+.002	5	4049.716
3658.285	+.005	1, n	3658.290	4050.465	+.002	3	4050.467

ZIRCONIUM — *continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
4054.579	+.002	3	4054.581	4183.459	+.013	4	4183.472
4055.171	+.002	5	4055.173	4186.821	+.013	2	4186.834
4055.849	+.002	4	4055.851	4191.635	+.015	2	4191.650
4056.653	+.002	1	4056.655	4191.931	+.016	1	4191.947
4057.982	+.002	2	4057.984	4194.147	+.016	2	4194.163
4058.769	+.002	2	4058.771	4194.909	+.016	4	4194.925
4060.231	+.002	1	4060.233	4196.280	+.017	3	4196.297
4060.728	+.002	2	4060.730	4227.839	+.041	8	4227.880
4061.676	+.002	4	4061.678	4231.644	+.039	4	4231.683
4064.301	+.002	8	4064.303	4231.716	+.039	3	4231.755
4068.870	+.002	2	4068.872	4234.717	+.038	3	4234.755
4071.240	+.002	2	4071.242	4236.153	+.037	5	4236.190
4072 840	+.002	10	4072.842	4237.517	+.037	2	4237.554
4075.070	+.002	3	4075.072	4239.392	+.036	8	4239.428
4076.676	+.002	3	4076.678	4240.420	+.034	8	4240.454
4077.199	+.002	2	4077.201	4241.286	+.033	8	4241.319
4078.455	+.002	4	4078.457	4241.770	+.033	8	4241.803
4081.359	+.002	10	4081.361	4253.660	+.030	2	4253.690
4082.439	+.002	2	4082.441	4256.546	+.029	2	4256.575
4083.239	+.002	2	4083.241	4258.142	+.029	7	4258.171
4084.450	+.002	3	4084.452	4261.304	+.027	2	4261.331
4085.838	+.002	5	4085.840	4261.526	+.027	2	4261.553
4087.836	+.002	2	4087.838	4264.115	+.026	1, n	4264.141
4090.662	+.002	6	4090.664	4265.015	+.026	1	4265.041
4090.943	+.002	3	4090.945	4266.828	+.025	1	4266.853
4093.311	+.002	2	4093.313	4268.116	+.025	5	4268.141
4094.416	+.002	2	4094.418	4273.620	+.023	4	4273.643
4096.781	+.002	3	4096.783	4274.861	+.023	2	4274.884
4099.458	+.002	2	4099 460	4277.465	+.022	2	4277.487
4107.690	+.002	3	4107.692	4282.306	+.020	5	4282.326
4108.544	+.002	3	4108.546	4285.354	+.019	1, n	4285.373
4110.195	+.002	1	4110.197	4286.615	+.019	1, n	4286.634
4110.801	+.002	1	4110.803	4289.266	+.018	1, n	4289.284
4113.115	+.002	1	4113.117	4290.314	+.018	1, n	4290.332
4120.309	+.002	1	4120.311	4291.454	+.016	2	4291.470
4121.601	+.002	5	4121.603	4294.897	+.017	7	4294.914
4128.125	+.002	1	4128.127	4296.311	+.016	2	4296.327
4135.828	+.002	4	4135.830	4300.681	+.015	1, n	4300.696
4140.158	+.003	2	4140.161	4302.990	+.015	5	4303.005
4146.034	+.004	1 n	4146.038	4304.803	+.014	3	4305.817
4149.339	+.004	10	4149.443	4306.034	+.014	1	4306.048
4151.118	+.005	4	4151.123	4309.931	+.013	1	4309.944
4152.788	+.005	4	4152.793	4312.342	+.012	1	4312.354
4156.377	+.006	8	4156.383	4317.424	+.011	5	4317.435
4161.345	+.007	7	4161.352	4319.164	+.011	2	4319.175
4166.501	+.008	4	4166.509	4321.287	+.011	2	4321.298
4169.494	+.009	1	4169.503	4324.145	+.011	2	4324.156
4171.616	+.009	3	4171.625	4325.554	+.010	4	4325.564
4179.953	+.011	3	4179.964	4329.685	+.010	2	4329.695
4182.710	+.012	5	4182.722	4333.380	+.010	2	4333.390

ZIRCONIUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
4336.463	+.010	1, N	4336.473	4490.392	+.010	1, N	4490.402
4337.741	+.010	1	4337.751	4494.550	+.010	3	4494.560
4341.248	+.010	7	4341.258	4495.588	+.010	1	4495.598
4342.356	+.010	1, n	4342.366	4497.139	+.010	7	4497.149
4343.160	+.010	1	4343.170	4507.250	+.010	4	4507.260
4343.517	+.010	1, n	4343.527	4526.265	+.010	1, n	4526.275
4346.641	+.010	2	4346.651	4535.877	+.010	5	4535.887
4347.349	+.010	1	4347.359	4542.351	+.010	4	4542.361
4347.469	+.010	1	4347.479	4550.260	+.011	2	4550.271
4348.009	+.010	8	4348.019	4553.142	+.011	1	4553.153
4358.870	+.010	1	4358.880	4554.100	+.011	1	4554.111
4359.852	+.010	8	4359.862	4554.174	+.011	2	4554.185
4360.427	+.010	4	4360.437	4555.659	+.011	2	4555.670
4366.571	+.010	4	4366.581	4558.175	+.011	1, n	4558.186
4371.078	+.010	7	4371.088	4565.587	+.012	1, n	4565.599
4373.203	+.010	1	4373.213	4574.633	+.013	1, n	4574.646
4379.899	+.010	7	4379.909	4582.435	+.014	1, n	4582.449
4389.746	+.010	1, n	4389.756	4590.675	+.016	1, n	4590.691
4395.066	+.010	3	4395.076	4614.092	+.004	1	4614.096
4400.365	+.010	1	4400.375	4629.223	+.004	1	4629.227
4401.481	+.010	1	4401.491	4634.139	+.004	1	4634.143
4403.472	+.010	1, n	4403.482	4640.290	+.004	1, n	4640.294
4414.439	+.010	2	4414.449	4644.982	+.004	1, n	4644.986
4414.665	+.010	3	4414.675	4657.795	+.004	1	4657.799
4420.588	+.010	3	4420.598	4661.958	+.004	1, n	4661.962
4427.373	+.010	2	4427.383	4667.314	+.004	1, n	4667.318
4429.235	+.010	1	4429.245	4683.592	+.004	1	4683.596
4431.619	+.010	3	4431.629	4687.971	+.004	3	4687.975
4435.976	+.010	1, n	4435.986	4688.621	+.004	1	4688.625
4436.900	+.010	1, n	4436.910	4707 952	+.002	1, n	4707.954
4454.929	+.010	5	4454.939	4710.250	+.002	3	4710.252
4455.564	+.010	1	4455.574	4712.085	+.002	1	4712.087
4456.428	+.010	1	4456.438	4717.795	.000	1, n	4717.795
4457.552	+.010	4	4457.562	4719.291	.000	1, n	4719.291
4460.485	+.010	4	4460.495	4732.510	—.003	1	4732.507
4460.920	+.010	1	4460.930	4739.655	—.004	1	4739.651
4467.044	+.010	1	4467.054	4762.955	—.008	1	4762.947
4468.354	+.010	1	4468.364	4772.498	—.009	2	4772.489
4469.654	+.010	1, n	4469.664	4785.103	—.009	1	4785.094
4470.451	+.010	2	4470.461	4788.862	—.009	1	4788.853
4470.688	+.010	4	4470.698	4806.060	—.009	1	4806.047†
4482.170	+.010	1	4482.180	4934.245	—.009	1	4934.236
4485.577	+.010	1, n	4485.587				

SOLAR LINES FOR STANDARDS.

PLATE 32.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3094.998	95.003‡	—.005	3247.667	.680*	—.013
3106.659	.677*	—.018	3260.373	.384‡	—.011
3115.152	.160†	—.008	3267.832	.839	—.007
3129.875	.882‡	—.005	3274.070	.092†	—.022
3137.443	.441†	+.002	3287.771	.791	—.020
3140.875	.870†	+.005	3292.153	.174*	—.021
3153.872	.870†	+.002	3295.932	.957	—.025
3167.294	.290‡	+.004	3302.485	.501‡	—.016
3172.174	.175†	—.001	3303.624	.648*	—.024
3176.103	.104†	—.001	3306.446	.471†	—.025
3188.165	.164‡	+.001	3308.907	.928*	—.021
3200.031	.032†	—.001	3318.131	.163	—.032
3218.390	.390	.000	3331.719	.741‡	—.022
3224.370	.368†	+.002	3347.990	48.011*	—.021
3231.437	.421*	+.016	3356.190	.222	—.032
3232.403	.404‡	—.001	3377.620	.667*	—.047
3646.123	.124	—.001	3389.847	.887	—.040

PLATE 36.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3441.103	.135*	—.032	3570.416	.402*	+.014
3444.026	.032*	—.006	3581.340	.344	—.004
3455.353	.384	—.031	3583.476	.483	—.007
3464.585	.609	—.024	3600.880	.880*	.000
3477.987	78.001	—.014	3606.829	.831*	—.002
3486.022	.036‡	—.014	3612.210	.217	—.007
3491.448	.464	—.016	3617.918	.920*	—.002
3500.695	.721‡	—.026	3618.922	.924*	—.002
3500.985	.993‡	—.008	3622.149	.149*	.000
3510.981	.987‡	—.006	3623.330	.332	—.002
3513.941	.947†	—.006	3623.601	.603	—.002
3518.482	.487	—.005	3623.603	.603	.000
3521.402	.404*	—.002	3628.846	.853*	—.007
3540.260	.266	—.006	3628.858	.853*	+.005
3545.335	.333	+.002	3631.624	.619*	+.005
3549.995	50.006*	—.011	3638.430	.435*	—.005
3564.673	.680*	—.007	3647.995	.995†	.000
3565.530	.528*	+.002	3652.690	.692‡	—.002

PLATE 36 — LANTHANUM—*continued.*

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3653.645	.639†	+.006	3749.620	.623*	—.003
3658.694	.688‖	+.006	3754.661	.664*	—.003
3667.397	.397‖	.000	3756.203	.211‡	—.008
3680.056	.064†	—.008	3758.364	.379*	—.015
3683.198	.202‖	—.004	3763.924	.442*	—.018
3687.595	.607*	—.012	3767.331	.344*	—.013
3695.188	.194‖	—.006	3770.116	.130*	—.014
3705.711	.711*	.000	3774.460	.486*	—.020
3707.185	.186‖	—.001	3780.826	.846‡	—.020
3710.429	.438*	—.009	3781.315	.330‡	—.015
3716.583	.585‖	—.002	3783.654	.674‡	—.020
3720.081	.086†	—.005	3788.016	.032*	—.016
3722.693	.691*	+.002	3793.995	94.014†	—.019
3727.758	.763*	—.005	3795.127	.150*	—.023
3735.020	.014*	+.006			

PLATE 40′ — LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3883.767	.773‡	—.006	4003.910	.916‡	—.006
3886.420	.427†	—.007	4016.578	.578‖	.000
3897.593	.599‡	—.006	4029.781	.796‖	—.015
3905.662	.666†	—.004	4030.900	.914*	—.014
3916.872	.875‡	—.003	4033.213	.225*	—.012
3924.664	.669‡	—.005	4034.623	.641*	—.018
3926.120	.123†	—.003	4035.864	.880*	—.016
3928.070	.071*	—.001	4045.960	.975*	—.015
3937.476	.474‡	+.002	4048.880	.893*	—.013
3941.019	.021‡	—.002	4055.688	.701‖	—.013
3944.145	.159*	—.014	4062.589	.602‖	—.013
3954.001	.001‡	.000	4063.741	.756*	—.015
3957.170	.180*	—.010	4071.882	.904*	—.022
3961.667	.676*	—.009	4073.901	.920‖	—.019
3971.465	.478‡	—.013	4077.860	.883*	—.023
3973.830	.835*	—.005	4083.739	.767*	—.c28
3977.887	.891‖	—.004	4088.702	.716‖	—.014
3981.916	.914‡*	+.002	4103.075	.101‖	—.026
3984.085	.078†	+.007	4107.626	.640*	—.014
3989.222	.216†	+.006	4114.578	.600‖	—.022

PLATE 40′′—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3950.101	.101	.000	4063.749	.756*	—.007
3954.005	.001	+.004	4071.893	.904*	—.011
3957.182	.180†	+.002	4073.918	.920*	—.002
3960.430	.429	+.001	4107.639	.646†	—.007
3971.478	.478†	.000	4114.602	.600	+.002
3984.085	.078†	+.007	4121.475	.481‡	—.006
3987.226	.216*	+.010	4157.923	.948	—.025
4003.918	.916‡	+.002	4185.036	.063	—.027
4016.577	.578	—.001	4197.224	.251†	—.027
4029.786	.796	—.010	4077.874	.883*	—.009
4030.900	.914†	—.014	4083.905	.928	—.023
4033.223	.225†	—.002	4088.710	.716	—.006
4034.632	.641†	—.009	4114.591	.600	—.009
4044.280	.293*	—.013	4157.918	.948	—.030
4045.980	.975*	+.005	4185.026	.063	—.037
4048.881	.893†	—.012	4197.222	.251†	—.029

PLATE 44.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4222.347	.381	—.034	4435.140	.132†	+.008
4254.498	.502*	—.004	4435.865	.852‡	+.013
4260.630	.638*	—.008	4447.909	.899‡	+.010
4293.232	.249†	—.017	4454.953	.950*	+.003
4318.811	.818†	—.007	4494.742	.735‡	+.007
4352.907	.903	+.004	4508.461	.456	+.005
4369.956	.943†	+.013	4554.210	.213†	—.003
4376.108	.103†	+.005	4571.273	.277‡	—.004
4407.850	.850†	.000	4578.733	.731	+.002
4425.624	.609‡	+.015			

PLATE 44′.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4250.258	.290*	—.032	4383.722	.721†	+.001
4250.945	.956†	—.011	4391.161	.149‡	+.012
4254.476	.502‡	—.026	4404.937	.927†	+.010
4260.623	.638*	—.015	4407.846	.850*	—.004
4267.928	.958*	—.030	4415.305	.299*	+.006
4271.914	.924†	—.010	4435.853	.852‡	+.001
4274.941	.958*	—.017	4435.133	.132*	+.001
4283.154	.170*	—.016	4447.912	.899‡	+.013
4289.510	.523*	—.013	4454.947	.950†	—.003
4289.872	.881*	—.009	4456.038	.047*	—.009
4293.231	.249*	—.018	4494.736	.735‖	+.001
4299.135	.152*	—.017	4497.031	.041‡	—.010
4302.681	.689*	—.008	4499.075	.070*	+.005
4307.889	.904*	—.015	4499.308	.315*	—.007
4308.057	.071*	—.014	4501.449	.444‖	+.005
4318.804	.818‖	—.014	4508.459	.456‖	+.003
4325.925	.940†	—.015	4554.206	.213‡	—.007
4343.400	.387*	+.013	4563.941	.939‖	+.002
4352.904	.903‖	+.001	4571.269	.277‖	—.008
4359.784	.778*	+.006	4572.146	.157‖	—.011
4369.946	.943‖	+.003	4578.724	.731‖	—.007
4376.117	.103‖	+.014	4588.368	.384‖	—.016

PLATE 44′′.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4215.617	.616†	+.001	4307.890	.904†	—.014
4226.860	.892*	—.032	4308.060	.034†	+.026
4250.268	.290†	—.022	4318.804	.818‡	—.014
4254.477	.502*	—.025	4325.927	.940†	—.013
4260.625	.638†	—.013	4359.786	.778†	+.008
4267.923	.958‡	—.035	4369.930	.943‡	—.013
4271.910	.924*	—.014	4376.102	.103‡	—.001
4274.947	.958†	—.011	4383.720	.721*	—.001
4283.555	.523*	+.032	4407.844	.850†	—.006
4283.142	.170*	—.038	4415.290	.299†	—.009
4289.511	.523*	—.012	4425.607	.609†	—.002
4293.231	.249†	—.018	4435.128	.132†	—.004
4299.144	.152*	—.008	4435.856	.852‡	+.004
4302.683	.689*	—.006	4447.894	.899‡	—.005
4306.057	.071†	—.014	4454.958	.950*	+.008

PLATE 44′′′.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4293.238	.249‡	—.011	4407.850	.850‡	.000
4318.814	.818†	—.004	4435.140	.132‡	+.008
4352.914	.903	+.011	4447.918	.899‡	+.019
4369.960	.943‡	+.017	4454.958	.950†	+.008
4376.115	.103‡	+.012	4494.750	.735‡	+.015
4425.623	.609‡	+.014	4508.468	.456‡	+.012
4435.871	.852‡	+.019	4554.218	.213†	+.005
4447.920	.899‡	+.021	4571.276	.277†	—.001

PLATE 48.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4629.505	.515‡	—.010	4824.335	.325‡	+.010
4637.669	.683	—.014	4810.723	.723*	.000
4643.631	.645	—.014	4903.483	.488†	—.005
4686.389	.395	—.006	4919.175	.183†	—.008
4703.999	.986	+.013	4924.955	.955	.000
4703.182	.180†	+.002	4934.244	.247†	—.003
4722.348	.349†	—.001	4973.262	.274	—.012
4727.638	.628‡	+.010	4978.760	.782†	—.022
4754.237	.226†	+.011	4994.301	.316	—.015
4859.930	.934‡	—.004			

PLATE 48′.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4602.152	.183	—.031	4783.607	.601*	+.006
4607.470	.509‡	—.039	4805.260	.253†	+.007
4611.423	.453‡	—.030	4810.734	.723*	+.011
4629.501	.515‡	—.014	4823.690	.697†	—.007
4637.672	.683	—.011	4824.337	.325‡	+.012
4643.625	.645	—.020	4859.936	.934	+.002
4648.821	.835†	—.014	4890.923	.945†	—.022
4668.289	.303†	—.014	4805.253	.253†	.000
4678.345	.353†	—.008	4810.728	.723*	+.005
4683.738	.743	—.005	4783.614	.601†	+.013
4686.398	.395	+.003	4754.222	.226‡	—.004
4690.312	.324	—.012	4727.620	.628‡	—.008
4691.567	.581†	—.014	4722.340	.349†	—.009
4691.570	.581†	—.011	4810.732	.723*	+.009
4703.186	.180†	+.006	4823.687	.697‡	—.010
4714.600	.599*	+.001	4824.329	.325‡	+.004
4722.341	.349‡	—.008	4859.938	.934‡	+.004
4727.634	.628†	+.006	4859.933	.934‡	—.001
4754.232	.226†	+.006	4900.091	.098‡	—.007

PLATE 52.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5005.851	.904‡	—.053	5225.691	.690‖	+.001
5006.251	.303‡	—.052	5230.020	.014‡	+.006
5014.379	.422*	—.043	5233.111	.124‡	+.013
5020.173	.210‡	—.037	5242.662	.662‡	.000
5049.972	50.008‖	—.036	5250.374	.391‡	—.017
5060.226	.252‡	—.026	5253.630	.649‖	—.019
5064.810	.833‡	—.023	5261.868	.880‡	—.012
5068.921	.946‖	—.025	5266.714	.729‡	—.015
5083.501	.525‡	—.024	5269.716	.722*	—.006
5090.933	.959‡	—.026	5273.548	.554*	—.006
5109.807	.825‡	—.018	5281.950	.968‡	—.018
5115.554	.558‡	—.024	5283.789	.803‡	—.014
5127.520	.530‡	—.010	5288.688	.708‡	—.020
5146.651	.664‡	—.013	5296.858	.873‡	—.015
5155.931	.937‡	—.006	5300.902	.918‡	—.016
5165.575	.588‡	—.013	5307.519	.546‡	—.027
5171.779	.783‡	—.004	5324.345	.373‡	—.028
5183.786	.792†	—.006	5333.060	.092‡	—.032
5189.020	.020†	.000	5349.617	.623†	—.006
5188.843	.863*	—.020	5361.775	.813†	—.038
5193.141	.139‡	+.002	5367.617	.670‡	—.053
5198.875	.885‡	—.010	5370.120	.165‡	—.045
5202.475	.483†	—.008	5379.724	.776‡	—.052
5204.700	.708†	—.008	5383.518	.576‡	—.058
5215.344	.352‡	—.008	5393.318	.378‡	—.060
5217.561	.559‡	+.002	5397.284	.346‡	—.062

PLATE 56.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5455.829	.826*	+.003	5590.345	.342†	+.003
5463.491	.493‖	—.002	5594.685	.695†	—.010
5466.601	.608‖	—.007	5598.519	.555*	—.036
5477.126	.128‖	—.002	5598.702	.715*	—.013
5487.959	.968‖	—.009	5601.501	.501‡	.000
5497.730	.731‖	—.001	5603.096	.097*	—.001
5501.683	.685‖	—.002	5615.525	.526‖	—.001
5513.204	.207‖	—.003	5615.882	.879‡	+.003
5528.631	.636†	—.005	5624.245	.253‖	—.008
5535.059	.073‖	—.014	5624.776	.768‖	+.008
5544.160	.158‖	+.002	5634.171	.167‖	+.004
5555.109	.113‖	—.004	5641.663	.661‖	+.002
5569.844	.848‡	—.004	5645.831	.835‖	—.004
5576.317	.319‖	—.002	5655.714	.707‖	+.007
5588.975	$.9_{80}$†	—.005	5658.073	.096*	—.023

PLATE 56.—LANTHANUM—*continued.*

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5662.744	.745	—.001	5742.059	.066	—.007
5675.648	.648	.000	5752.246	.257	—.011
5679.245	.249	—.004	5754.869	.884‡	—.015
5682.859	.861	—.002	5763.205	.215†	—.010
5688.431	.434‡	—.003	5772.358	.360‡	—.002
5701.772	.769‡	+.003	5782.342	.346†	—.004
5708.614	.620†	—.006	5784.066	.081	—.015
5711.304	.318*	—.014	5788.123	.136‡	—.013
5715.300	.309‡	—.009	5791.192	.207*	—.015
5731.980	.973‡	+.007	5798.056	.087	—.031

PLATE 54.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5784.074	.081	—.007	5884.062	.048*	+.014
5788.130	.136	—.006	5905.895	.895‡	.000
5798.071	.087	—.016	5914.379	.384†	—.005
5798.391	.400‡	—.009	5916.480	.475‡	+.005
5806.950	.954‡	—.004	5919.853	.855‡	—.002
5809.438	.437‡	+.001	5930.405	.410‡	—.005
5816.590	.594†	—.004	5934.885	.883‡	+.002
5831.820	.832	—.012	5948.761	.761‡	.000
5853.898	.903	—.005	5956.923	.925‡	—.002
5857.666	.672†	—.006	5975.570	.576	—.006
5862.585	.580†	+.005			

THE ARC-SPECTRUM OF LANTHANUM.

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
3104.694	+.008	1	3104.702	4031.846		7	4031.847†
3142.882	—.001	2	3142.881	4035.328	.000	1, n	4035.328
3176.105	—.002	2	3176.103	4035.878	.000	1, n	4035.878
3215.936	—.001	2	3215.935	4036.976		1, n	4036.974†
3236.670	+.001	1	3236.671	4043.071		8	4043.070†
3245.247	+.003	5	3245.250	4045.961	+.001	10	4045.962
3249.477	+.004	4	3249.481	4050.215		6	4050.217†
3265.786	+.009	5	3265.795	4059.631	+.002	1, n	4059.633
3303.219	+.022	5	3303.241	4060.460		4	4060.459†
3337.597	+.033	6	3337.630	4062.886	+.002	1, n	4062.888
3344.671	+.034	5	3344.705	4063.732	+.003	1	4063.735
3376.436	+.036	3	3376.472	4064.922		3	4064.922†
3381.010	+.036	8	3381.046	4065.713		3	4065.715†
3452.296	+.034	3	3452.330	4067.520		5	4067.519†
3453.279	+.033	3	3453.312	4076.841	+.004	2	4076.845
3461.300	+.027	3	3461.327	4077.499	+.004	10	4077.503
3510.116	+.005	3	3510.121	4077.852	+.004	1	4077.856
3513.045	+.005	3	3513.050	4078.886	+.004	1	4078.890
3514.187	+.004	4	3514.191	4086.843	+.005	10	4086.848
3574.560	+.005	5	3574.565	4089.746	+.006	1	4089.752
3641.677	—.002	5	3641.675	4090.541	+.006	1	4090.547
3645.549	—.002	5	3645.547	4090.161	+.006	1	4090.167
3649.664	—.003	5	3649.661	4090.914	+.006	6	4090.920
3650.316	—.003	5	3650.313	4095.271	+.007	3	4095.278
3672.150	—.003	3	3672.147	4099.671		8	4099.678†
3680.050	—.002	1	3680.048	4105.018		10	4105.026†
3734.995	+.006	3	3735.001	4123.382	+.012	3	4123.394
3759.205	+.012	7	3759.217	4181.856		10	4141.872†
3790.935	+.018	6	3790.953	4144.075	+.017	2	4144.092
3794.886	+.018	7	3793.904	4152.098	+.021	8	4152.119
3883.948	+.007	2	3883.955	4152.100	+.020	2	4154.120
3886.489		7	3886.495†	4192.465	+.036	8	4192.501
3916.050	+.003	4	3916.053	4194.332	+.037	1	4194.369
3936.351	.000	7	3936.351	4194.617		1	4194.654†
3949.256	.000	6	3949.256	4204.155	+.063	5	4204.218
3988.669	.000	5	3988.669	4215.637	+.046	2	4215.683
3995.903	.000	5	3995.903	4217.690	+.043	6	4217.733
4013.400	—.001	1	4013.399	4226.862	+.036	6	4226.898
4013.535	.000	1	4013.535	4231.074	+.033	4	4231.107
4015.535		2	4015.531†	4238.512		10	4238.543†
4023.720		3	4023.717†	4250.126		5	4250.144†
4023.999	.000	4	4023.999	4254.478	+.026	1	4254.504
4025.786	.000	3	4025.786	4263.734		7	4263.742†
4026.014		7	4026.013†	7275.782		4	4275.797†

† Mean of several values obtained from different plates.

LANTHANUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
4280.411		3	4280.418†	4692.678	+.002	6	4692.680
4282.537		2	4282.560†	4699.810	+.001	1, n	4699.811
4283.165	+.015	1	4283.180	4703.458	+.001	1	4703.459
4287.111		10	4287.128†	4703.429	.000	1	4703.429
4296.196		10	4296.210†	4716.624	—.001	3	4716.623
4300.765		2	4300.782†	4716.705	—.001	1, n	4716.704
4302.683		1	4302.699†	4728.598		5	4728.594†
4316.060		8	4316.076†	4743.277		7	4743.273†
4318.813	+.012	1	4318.825	4748.915		4	4748.911†
4333.924		15	4333.934†	4767.078	—.005	2	4767.073
4335.126		7	4335.134†	4796.868		1, n	4796.862†
4354.565		8	4354.565†	4804.221		4	4804.219†
4363.223		3	4363.221†	4809.188		5	4809.183†
4378.282		8	4378.274†	4809.695	—.005	5	4809.690
4383.628		7	4383.624†	4824.243		5	4824.239†
4385.360		4	4385.364†	4839.703		2	4839.697†
4411.380		2	4411.379†	4840.207		3	4840.203†
4419.332		3	4419.329†	4850.775		1, n	4850.772†
4423.350		1, n	4423.354†	4851.000	—.000	1, n	4851.000
4424.083		4	4424.082†	4861.082		4	4861.081†
4425.595	+.003	1, n	4425.598	4900.099	—.003	6	4900.096
4427.736		8	4427.741†	4921.148	+.001	6	4921.149
4430.079		10	4430.075†	4921.978	+.001	6	4921.979
4432.295	+.001	1, n	4432.296	4935.000	+.003	3	4935.003
4435.141		1	4435.139†	4949.941	+.006	3	4949.947
4435.389	+.003	1, n	4435.392	4952.232	+.006	1	4952.238
4452.331		5	4452.327†	4970.556	+.010	5	4970.566
4455.968		5	4455.965†	4987.044	+.015	6	4987.059
4499.223		2	4499.223†	4991.436	+.016	3	4991.452
4522.540		10	4522.544†	4999.625	+.017	0	4999.642
4525.468	—.002	7	4525.466	5001.929	+.050	11, n	5001.979
4526.279	+.014	8	4526.293	5002.255	+.050	1, n	5002.305
4549.677		7	4549.679†	5047.019	+.032	1, n	5047.051
4550.338		1	4550.337†	5050.704	+.030	.1	5050.734
4550.952		2	4550.956†	5056.600	+.028	1	5056.628
4558.653		7	4558.660†	5063.071	+.026	1, n	5063.097
4568.085		5	4568.094†	5106.380	+.015	1	5106.395
4570.202		5	4570.215†	5114.709	+.014	4	5114.723
4575.048		8	4575.059†	5123.137	+.012	4	5123.149
4580.234		5	4580.245†	5145.573	+.009	1	5145.582
4605.922	+.029	4	4605.951	5156.881	+.007	1	5156.888
4613.532	+.025	5	4613.557	5157.575	+.007	1	5157.582
4620.022		3	4620.054†	5158.839	+.007	1	5185.846
4655.650		8	4655.667†	5163.762	+.006	1	5163.768
4662.661		5	4662.678†	5177.448	+.005	3	5177.453
4663.936	+.007	5	4663.943	5183.559	+.005	0, d	5183.664
4669.076		5	4669.080†	5188.366	+.005	11	5188.371
4671.985		4	4671.994†	5204.289	+.004	1	5204.293
4688.820		1, n	4688.824†	5212.010	+.004	1, n	5212.014
4691.349		4	4691.364†	5234.430	+.005	2	5234.435

LANTHANUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
5259.531	+.008	2	5259.539	5565.928	+.003	3	5565.931
5271.327	+.010	2	5271.337	5568.671	+.003	5	5568.674
5290.973	+.015	3	5290.988	5588.555	+.003	3	5588.558
5302.116	+.019	3	5302.135	5657.939	+.001	1	5657.940
5302.774	+.019	2	5302.793	5671.765	.000	2	5671.765
5303.686	+.022	3	5303.708	5703.530	.000	1	5703.530
5340.803	+.033	1	5340.836	5720.221	+.002	2, n	5720.223
5358.003	+.041	1, n	5358.044	5735.155	+.004	2, n	5735.159
5377.213	+.052	2	5377.265	5740.866	+.005	4	5740.871
5381.114	+.054	2	5381.168	5744.619	+.006	4	5744.625
5381.911	+.055	1, n	5381.966	5762.034	+.010	4	5762.044
5382.051	+.055	2	5382.106	5769.273	+.012	5	5769.285
5455.348	+.003	8	5455.351	5769.533	+.012	5	5769.545
5458.884	+.003	1	5458.887	5789.427	+.011	6	5789.438
5464.571	+.003	2	5464.574	5791.528	+.011	4	5791.539
5475.448	+.002	1, n	5475.450	5797.776	+.009	4	5795.785
5480.938	+.002	1, n	5480.940	5805.976	+.008	2	5805.984
5482.472	+.002	2	5482.474	5808.517	+.007	2	5808.524
5491.275	+.002	1	5491.277	5822.180	+.005	1	5822.185
5493.654	+.002	2	5493.656	5824.032	+.005	1	5824.037
5501.557	+.002	8	5501.559	5829.926	+.003	1, n	5829.929
5502.463	+.002	2	5502.465	5845.242	+.001	1, n	5845.243
5502.876	+.002	2	5502.878	5848.583	+.001	1, n	5848.584
5504.019	+.002	4	5504.021	5855.792	.000	1	5855.792
5515.500	+.002	1	5515.502	5863.903	+.001	2	5863.902
5517.560	+.002	2	5517.562	5874.202	+.002	1, n	5874.200
5535.886	+.003	3	5535.889	5874.943	+.002	1, n	5874.941
5545.129	+.003	2	5545.132	5930.330	.000	1	5830.330
5565.657	+.003	5	5565.660				

Minor Contributions and Notes.

WILLIAM A. ROGERS.

On March 1, of the present year, in the city of Waterville, Me., the seat of Colby University, William Augustus Rogers passed away. He had been engaged, even up to a few weeks before his death, in teaching duties, and in investigations connected with standards of length.

He was born in Waterford, Conn., November 13, 1832, prepared for college at Alfred Academy, New York, and graduated from Brown University in 1857. For the next thirteen years he was connected with Alfred Academy, first as tutor, later as professor of mathematics and astronomy. During this time, however, he was allowed to spend one year as student of theoretical and applied mechanics in the Sheffield Scientific School of Yale College, and one year as student of astronomy in the Harvard College Observatory. In 1870 he was appointed assistant in the Observatory, and from 1877 to 1886 he filled the position of assistant professor of astronomy.

During his stay at Harvard Observatory he was engaged in observing and mapping out all the stars down to the ninth magnitude in a belt of five degrees a little north of our zenith. This work formed part of the German survey of the northern heavens under the auspices of the Astronomische Gesellschaft. Five volumes containing the results of this work have been published, and one is in process of compilation.

About this time he became interested in the construction of comparators. In connection with George M. Bond, of Hartford, Conn., he designed the Rogers-Bond universal comparator. One of these instruments was used by the Pratt and Whitney Company in establishing their system of standard gauges, and another was used by Professor Rogers in Waterville in the comparison of standards of length.

In 1879 the American Academy of Arts and Sciences sent Professor Rogers to Europe to obtain authorized copies of the Imperial yard and the mètre des archives. The copies so obtained have been used in the comparisons of yard and meter bars made by him for the Depart-

ment of Standards of the British Board of Trade, the United States Signal Service, the Lick Observatory, and for most of the large universities of the United States and Canada. A paper by Professor Rogers presented to the A. A. A. S. in 1880, "On the Present State of the Question of the Standards of Length," was at its time an exceedingly important contribution.

His work on standards led him to the consideration of various dependent problems, such as the action of a diamond in ruling lines on glass, the radiation of heat between metals, and the practical solution of the perfect screw problem. His contributions to the literature and data of these problems were important. As a result of his work we have the Rogers-Ballou process of cutting a screw. His very perfect dividing engines are the embodiment of his principles.

In 1886 Professor Rogers accepted the chair of Physics and Astronomy in Colby University. The Shannon Physical Laboratory, designed by him, was specially suited for constant temperature work. Here he carried on his work of the comparison of bars.

In 1890 Professor Morley became associated with Professor Rogers in the determination of the absolute expansion of metal bars in wavelengths of sodium light. The investigations were carried on in the Shannon Laboratory. Though results were obtained for only a few bars, these went to show that the experimental difficulties had at last been gotten rid of and that the accuracy of the method was all that could be desired.

Professor Rogers was the recipient of many honors. In 1880 he was made a Fellow of the Royal Society of England, and five years later was elected an Honorary Fellow of that body. He was also an Honorary Fellow of the Royal Microscopical Society; a Fellow of the American Association for the Advancement of Science; twice vice president of Section A, and once of Section B, of the A. A. A. S.; was one of the hundred members of the American Academy of Arts and Sciences, and was also a member of the National Academy of Sciences. He received the honorary degrees of A.M., Ph.D., LL.D., from Yale, Alfred, and Brown Universities respectively.

In disposition, Professor Rogers was kindly and generous. His broad sympathies led him to take an active and intelligent interest in civic and religious life. He was honored and loved by the student, and his co-workers on the faculty. In 1857, just after his graduation from Brown, he married Rebecca Jane Titsworth who, with two sons, Fred.

P. Rogers, M.D., of Providence, R. I., and Arthur K. Rogers, Fellow in Philosophy in the University of Chicago, survives him.

G. F. H.

PHOTOGRAPHIC SPECTRUM OF THE AURORA.[1]

VARIOUS attempts have been made at this Observatory to photograph the spectrum of the aurora. In 1886 on several occasions long exposures were given to plates during bright auroras, but no result was obtained. On April 1, 1897, Mr. Edward S. King succeeded in obtaining a photograph in which four bright lines were visible, but uncertainty existed regarding their wave-lengths. The exposure was 147 minutes. During the bright aurora of March 15, 1898, he obtained a photograph showing two bright lines. The exposure was 141 minutes. The brightest of these lines extends in wave-length from about 3892 to 3925, and the wave-length of the second is 4285. Assuming the two brighter lines photographed in 1897 to be identical with these, the four lines on that plate have the wave-lengths 3862, 3922, 4288 and 4694. The first of these lines is very faint.

The errors of measurement of these lines do not exceed one or two units, but much greater uncertainty exists in the reduction owing to difficulties in comparing them with the lines of the solar spectrum which was photographed upon the same plate. Probably the two auroras gave different spectra. That in 1897 was taken with a wide slit, but the images of the lines were well defined on the edges and of equal width, so that the line 3922 was probably really narrow and coincident with the edge of greater wave-length of the line 3892 to 3925. The spectroscope used was not especially designed for photographing faint surfaces and it is hoped that better results may be obtained with a new instrument now in course of construction. As is the case with all results announced in these *Circulars* it is expected that full details will be published later in the *Annals* of the Observatory.

EDWARD C. PICKERING.

March 23, 1898.

[1] *Harvard College Observatory, Circular* No. 28.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation,

It is intended to publish from time to time a bibliography of astrophysics, in which will be found the titles of recently published astrophysical and spectroscopic papers. In order that this list may be as complete as possible, and that current work in astrophysics may receive appropriate notice in other departments of the JOURNAL, authors are requested to send copies of all papers on these and closely allied subjects to both Editors

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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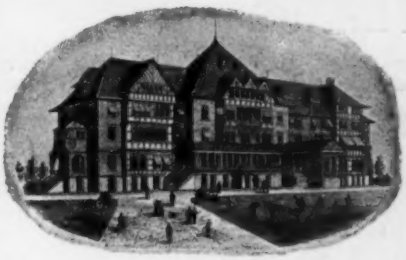

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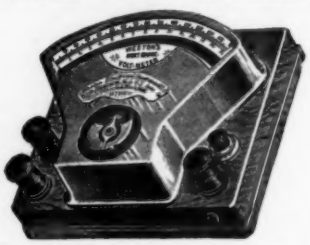

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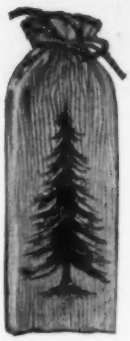

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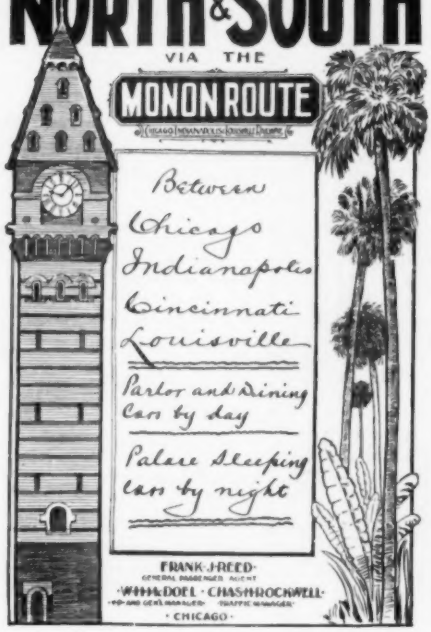

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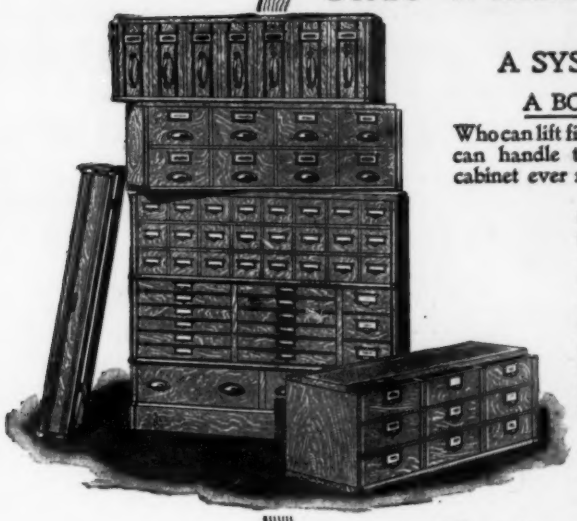

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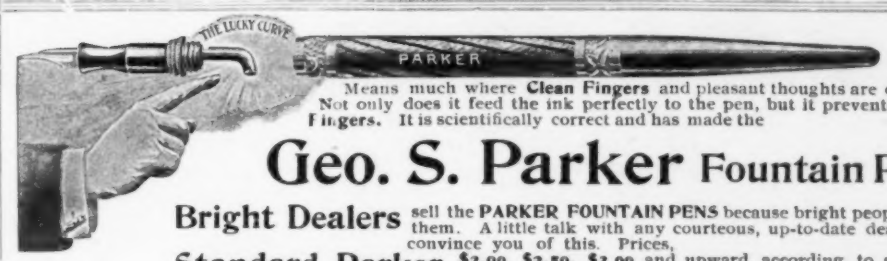

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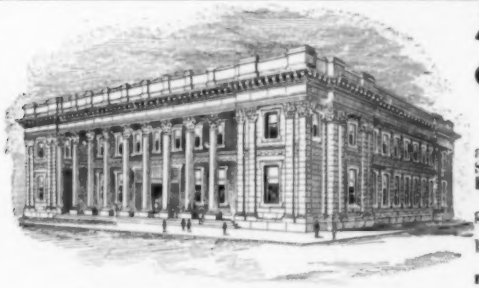

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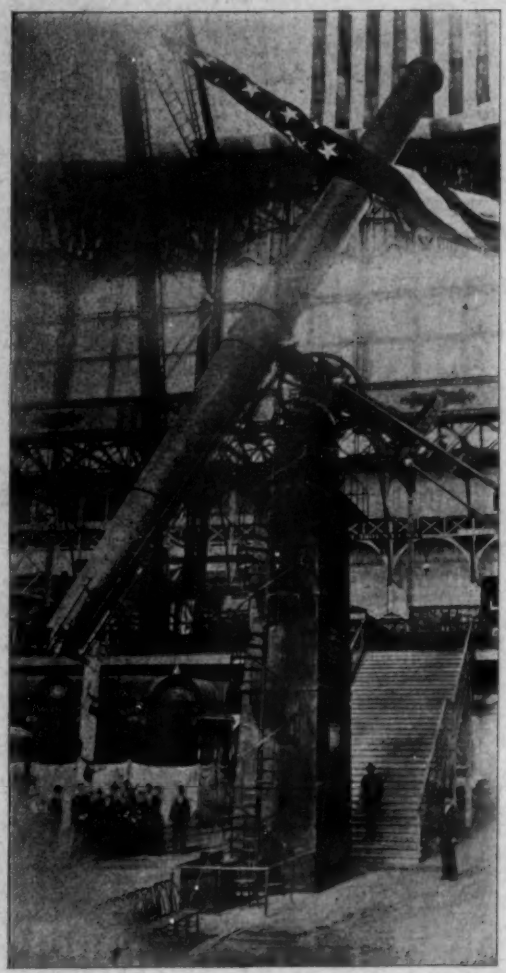

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